Verifying Trigonometric Identities

Name: Key

Verify each identity.

1. $\frac{\sec\theta+1}{\tan\theta} = \frac{\tan\theta}{\sec\theta-1}$

$\frac{\sec\theta+1}{\sec\theta+1}$

$= \frac{\tan\theta(\sec\theta+1)}{\sec^2\theta-1}$

$= \frac{\tan\theta(\sec\theta+1)}{\tan^2\theta}$

$\frac{\sec\theta+1}{\tan\theta} = \frac{\sec\theta+1}{\tan\theta}$

2. $\sin^2A\cot^2A = (1-\sin A)(1+\sin A)$

$\sin^2A\frac{\cos^2A}{\sin^2A} = 1-\sin^2A$

$\cos^2A = \cos^2A$

3. $\frac{1}{1-\cos x}+\frac{1}{1+\cos x} = 2\csc^2x$

$\frac{1+\cos x+1-\cos x}{1-\cos^2x} = 2\csc^2x$

$\frac{2}{\sin^2x} = 2\csc^2x$

$2\csc^2x = 2\csc^2x$

4. $(\cot^2x+1)(\sin^2x-1) = -\cot^2x$

$\csc^2x(-\cos^2x) = -\cot^2x$

$\frac{1}{\sin^2x}(-\cos^2x) = -\cot^2x$

$\frac{-\cos^2x}{\sin^2x} = -\cot^2x$

$-\cot^2x = -\cot^2x$

5. $\csc x+\cot x = \frac{\sin x}{1-\cos x}$

$\frac{1}{\sin x}+\frac{\cos x}{\sin x} =$

$\frac{(1-\cos x)}{(1-\cos x)}\frac{1+\cos x}{\sin x} =$

$\frac{1-\cos^2x}{(1-\cos x)\sin x} =$

$\frac{\sin^2x}{(1-\cos x)\sin x} = \frac{\sin x}{1-\cos x}$

$\frac{\sin x}{1-\cos x} = \frac{\sin x}{1-\cos x}$

6. $\sin^2x(1+\cot^2x) = 1$

$\sin^2x+\sin^2x\frac{\cos^2x}{\sin^2x} = 1$

$\sin^2x+\cos^2x = 1$

$1 = 1$ ✓

7. $\tan x\csc x\cos x = 1$

$\frac{\sin x}{\cos x}\cdot\frac{1}{\sin x}\cdot\frac{\cos x}{1} = 1$

$1 = 1$

8. $\cos x+\sin x\tan x = \sec x$

$\cos x+\sin x\frac{\sin x}{\cos x} = \sec x$

$\frac{\cos x}{\cos x}\cdot\cos x+\frac{\sin^2x}{\cos x}$

$\frac{\cos^2x}{\cos x}+\frac{\sin^2x}{\cos x} = \sec x$

$\frac{\cos^2x+\sin^2x}{\cos x} = \sec x$

$\frac{1}{\cos x} = \sec x$

$\sec x = \sec x$

9. $\frac{\cos^2x}{1+\sin x} = 1-\sin x$

$\frac{1-\sin x}{1-\sin x}$

$\frac{(1-\sin x)(\cos^2x)}{1-\sin^2x} = 1-\sin x$

$\frac{(1-\sin x)(\cos^2x)}{\cos^2x} = 1-\sin x$

$1-\sin x = 1-\sin x$

Simplify each expression.

11. $\frac{\csc^2 x - 1}{\csc^2 x}$

$\frac{\cot^2 x}{\csc^2 x} = \frac{\frac{\cos^2 x}{\sin^2 x}}{\frac{1}{\sin^2 x}}$

$= \frac{\cos^2 x}{\sin^2 x} \cdot \frac{\sin^2 x}{1}$

$= \boxed{\cos^2 x}$

12. $\frac{\sec x}{\sin x} - \frac{\sin x}{\cos x}$

$\frac{\cos x}{\cos x} \cdot \frac{\frac{1}{\cos x}}{\sin x} - \frac{\sin x}{\cos x} \cdot \frac{\sin x}{\sin x}$

$\frac{1}{\cos x \sin x} - \frac{\sin^2 x}{\cos x \sin x} = \frac{1 - \sin^2 x}{\cos x \sin x}$

$= \frac{\cos^2 x}{\cos x \sin x} = \frac{\cos x}{\sin x} = \boxed{\cot x}$

13. $\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$, $\frac{\sin x}{\sin x}$, $\frac{\cos x}{\cos x}$

$\frac{\sin^2 x}{\cos x \sin x} + \frac{\cos^2 x}{\cos x \sin x}$

$\frac{\sin^2 x + \cos^2 x}{\cos x \sin x} = \frac{1}{\cos x} \cdot \frac{1}{\sin x}$

$= \boxed{\sec x \cdot \csc x}$

14. $\frac{\sec x + \tan x}{\sec x}$

$\frac{\sec x}{\sec x} + \frac{\tan x}{\sec x}$

$1 + \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} = 1 + \frac{\sin x}{\cos x} \cdot \frac{\cos x}{1}$

$= \boxed{1 + \sin x}$

15. $\sin x + \sin x \cot^2 x$

$\sin x (1 + \cot^2 x)$

$\sin x (\csc^2 x)$

$\frac{1}{\csc x} \cdot \csc^2 x = \boxed{\csc x}$

Find a numerical value of one trigonometric function of x.

16. $2\cot x \cos x + 2\sin x = \sec x$

$2 \cdot \frac{\cos x}{\sin x} \cdot \frac{\cos x}{1} + 2\sin x = \sec x$

$2\left(\frac{\cos^2 x}{\sin x} + \frac{\sin^2 x}{\sin x}\right) = \sec x$

$\sin x \cdot \frac{2}{\sin x} = \sec x \cdot \sin x$

$2 = \frac{1}{\cos x} \cdot \sin x$ $\boxed{2 = \tan x}$

17. $\cot x + \sin x = -\cos x \cot x$

$\frac{\cos x}{\sin x} + \frac{\sin^2 x}{\sin x} = -\cos x \frac{\cos x}{\sin x}$

$\frac{\cos x + \sin^2 x}{\sin x} = \frac{-\cos^2 x}{\sin x}$

$+ \frac{\cos^2 x}{\sin x}$ $+ \frac{\cos^2 x}{\sin x}$

$\frac{\cos x + \sin^2 x + \cos^2 x}{\sin x} = 0$

$\frac{1 + \cos x}{\sin x} = 0$

$1 + \cos x = 0$

$\boxed{\cos x = -1}$

18. $\tan x = \frac{1}{4}\sec x$

$\frac{\tan x}{\sec x} = \frac{\frac{1}{4}\sec x}{\sec x}$

$\frac{\tan x}{\sec x} = \frac{1}{4}$

$\frac{\sin x}{\cos x} \cdot \frac{\cos x}{1} = \frac{1}{4}$

$\boxed{\sin x = \frac{1}{4}}$