5-7

NAME ______________________ DATE ______________ PERIOD ________

Study Guide

The Ambiguous Case for the Law of Sines

If we know the measures of two sides and a nonincluded angle of a triangle, three situations are possible: no triangle exists, exactly one triangle exists, or two triangles exist. A triangle with two solutions is called the **ambiguous case**.

Case 1: $A < 90°$ for a, b, and A	
$a < b \sin A$	no solution
$a = b \sin A$	one solution
$a \geq b$	one solution
$b \sin A < a < b$	two solutions
Case 2: $A \geq 90°$	
$a \leq b$	no solution
$a > b$	one solution

Example **Find all solutions for the triangle if $a = 20$, $b = 30$, and $A = 40°$. If no solutions exist, write *none*.**

Since $40° < 90°$, consider Case 1.
$b \sin A = 30 \sin 40°$
$b \sin A \approx 19.28362829$
Since $19.3 < 20 < 30$, there are two solutions for the triangle.
Use the Law of Sines to find B.

$$\frac{20}{\sin 40°} = \frac{30}{\sin B} \qquad \frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\sin B = \frac{30 \sin 40°}{20}$$

$$B = \sin^{-1}\left(\frac{30 \sin 40°}{20}\right)$$

$$B \approx 74.61856831$$

So, $B \approx 74.6°$. Since we know there are two solutions, there must be another possible measurement for B. In the second case, B must be less than 180° and have the same sine value. Since we know that if $\alpha < 90$, $\sin \alpha = \sin (180 - \alpha)$, $180° - 74.6°$ or $105.4°$ is another possible measure for B. Now solve the triangle for each possible measure of B.

Solution I

$C \approx 180° - (40° + 74.6°)$ or $65.4°$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{20}{\sin 40°} \approx \frac{c}{\sin 65.4°}$$

$$c \approx \frac{20 \sin 65.4°}{\sin 40°}$$

$$c \approx 28.29040558$$

One solution is $B \approx 74.6°$, $C \approx 65.4°$, and $c \approx 28.3$.

Solution II

$C \approx 180° - (40° + 105.4°)$ or $34.6°$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{20}{\sin 40°} \approx \frac{c}{\sin 34.6°}$$

$$c \approx \frac{20 \sin 34.6°}{\sin 40°}$$

$$c \approx 17.66816088$$

Another solution is $B \approx 105.4°$, $C \approx 34.6°$, and $c \approx 17.7$.