

27. $\sin x + \cos x \stackrel{?}{=} \frac{\cos x}{1 - \tan x} + \frac{\sin x}{1 - \cot x}$

$$\begin{aligned}\sin x + \cos x &\stackrel{?}{=} \frac{\cos x}{1 - \frac{\sin x}{\cos x}} + \frac{\sin x}{1 - \frac{\cos x}{\sin x}} \\ \sin x + \cos x &\stackrel{?}{=} \frac{\cos x}{1 - \frac{\sin x}{\cos x}} \cdot \frac{\cos x}{\cos x} + \frac{\sin x}{1 - \frac{\cos x}{\sin x}} \cdot \frac{\sin x}{\sin x} \\ \sin x + \cos x &\stackrel{?}{=} \frac{\cos^2 x}{\cos x - \sin x} + \frac{\sin^2 x}{\sin x - \cos x} \\ \sin x + \cos x &\stackrel{?}{=} -\frac{\cos^2 x}{\sin x - \cos x} + \frac{\sin^2 x}{\sin x - \cos x} \\ \sin x + \cos x &\stackrel{?}{=} \frac{\sin^2 x - \cos^2 x}{\sin x - \cos x} \\ \sin x + \cos x &\stackrel{?}{=} \frac{(\sin x + \cos x)(\sin x - \cos x)}{\sin x - \cos x} \\ \sin x + \cos x &= \sin x + \cos x\end{aligned}$$

28. $\sin \theta + \cos \theta + \tan \theta \sin \theta \stackrel{?}{=} \sec \theta + \cos \theta \tan \theta$

$$\begin{aligned}\sin \theta + \cos \theta + \frac{\sin \theta}{\cos \theta} \sin \theta &\stackrel{?}{=} \sec \theta + \cos \theta \tan \theta \\ \sin \theta + \cos \theta + \frac{\sin^2 \theta}{\cos \theta} &\stackrel{?}{=} \sec \theta + \cos \theta \tan \theta \\ \sin \theta + \frac{\cos^2 \theta}{\cos \theta} + \frac{\sin^2 \theta}{\cos \theta} &\stackrel{?}{=} \sec \theta + \cos \theta \tan \theta \\ \sin \theta + \frac{\cos^2 \theta + \sin^2 \theta}{\cos \theta} &\stackrel{?}{=} \sec \theta + \cos \theta \tan \theta \\ \sin \theta + \frac{1}{\cos \theta} &\stackrel{?}{=} \sec \theta + \cos \theta \tan \theta \\ \sin \theta + \sec \theta &\stackrel{?}{=} \sec \theta + \cos \theta \tan \theta \\ \sin \theta \frac{\cos \theta}{\cos \theta} + \sec \theta &\stackrel{?}{=} \sec \theta + \cos \theta \tan \theta \\ \cos \theta \frac{\sin \theta}{\cos \theta} + \sec \theta &\stackrel{?}{=} \sec \theta + \cos \theta \tan \theta \\ \cos \theta \tan \theta + \sec \theta &\stackrel{?}{=} \sec \theta + \cos \theta \tan \theta \\ \sec \theta + \cos \theta \tan \theta &= \sec \theta + \cos \theta \tan \theta\end{aligned}$$

29. Sample answer: $\sec x = \sqrt{2}$

$$\begin{aligned}\frac{\csc x}{\cot x} &= \sqrt{2} \\ \frac{1}{\sin \theta} \frac{\cos \theta}{\cos \theta} &= \sqrt{2} \\ \frac{1}{\sin \theta} &= \sqrt{2} \\ \frac{1}{\cos x} &= \sqrt{2} \\ \sec x &= \sqrt{2}\end{aligned}$$

30. Sample answer: $\tan x = 2$

$$\begin{aligned}\frac{1 + \tan x}{1 + \cot x} &= 2 \\ \frac{1 + \frac{\sin x}{\cos x}}{1 + \frac{\cos x}{\sin x}} &= 2 \\ \frac{\cos x + \sin x}{\sin x + \cos x} &= 2 \\ \frac{\cos x}{\sin x} &= 2 \\ \frac{\sin x}{\cos x} &= 2 \\ \tan x &= 2\end{aligned}$$

31. Sample answer: $\cos x = 0$

$$\begin{aligned}\frac{1}{\cot x} - \frac{\sec x}{\csc x} &= \cos x \\ \tan x - \frac{\cos x}{\frac{1}{\sin x}} &= \cos x \\ \tan x - \frac{\sin x}{\cos x} &= \cos x \\ \tan x - \tan x &= \cos x \\ 0 &= \cos x\end{aligned}$$

32. Sample answer: $\sin x = \frac{1}{2}$

$$\begin{aligned}\frac{1 + \cos x}{\sin x} + \frac{\sin x}{1 + \cos x} &= 4 \\ \frac{1 + 2 \cos x + \cos^2 x}{\sin x(1 + \cos x)} + \frac{\sin^2 x}{\sin x(1 + \cos x)} &= 4 \\ \frac{1 + 2 \cos x + \cos^2 x + \sin^2 x}{\sin x(1 + \cos x)} &= 4 \\ \frac{2 + 2 \cos x}{\sin x(1 + \cos x)} &= 4 \\ \frac{2(1 + \cos x)}{\sin x(1 + \cos x)} &= 4 \\ \frac{2}{\sin x} &= 4 \\ 2 &= 4 \sin x \\ \frac{1}{2} &= \sin x\end{aligned}$$

33. Sample answer: $\sin x = 1$

$$\begin{aligned}\cos^2 x + 2 \sin x - 2 &= 0 \\ 1 - \sin^2 x + 2 \sin x - 2 &= 0 \\ 0 &= \sin^2 x - 2 \sin x + 1 \\ 0 &= (\sin x - 1)^2 \\ 0 &= \sin x - 1 \\ \sin x &= 1\end{aligned}$$

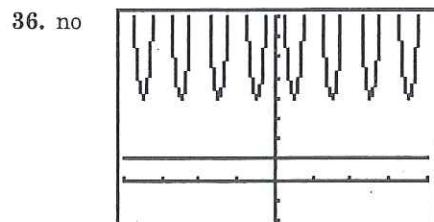
34. Sample answer: $\cot x = 1$

$$\begin{aligned}\csc x &= \sin x \tan x + \cos x \\ \csc x &= \sin x \frac{\sin x}{\cos x} + \cos x \\ \csc x &= \frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x} \\ \csc x &= \frac{1}{\cos x} \\ \frac{1}{\sin x} &= \frac{1}{\cos x} \\ \frac{\cos x}{\sin x} &= 1 \\ \cot x &= 1\end{aligned}$$

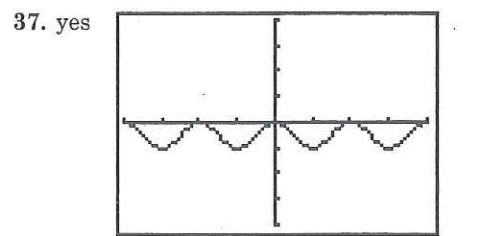
35. $\frac{\tan^3 \theta - 1}{\tan \theta - 1} - \sec^2 \theta - 1 = 0$

$$\begin{aligned}\frac{(\tan \theta - 1)(\tan^2 \theta + \tan \theta + 1)}{\tan \theta - 1} - (\tan^2 \theta + 1) - 1 &= 0 \\ \tan^2 \theta + \tan \theta + 1 - \tan^2 \theta - 1 - 1 &= 0 \\ \tan \theta - 1 &= 0 \\ \tan \theta &= 1\end{aligned}$$

$$\begin{aligned}\cot \theta &= \frac{1}{\tan \theta} \\ \cot \theta &= \frac{1}{1} \\ \cot \theta &= 1\end{aligned}$$



$[-2\pi, 2\pi]$ scl: $\frac{\pi}{2}$ by $[-2, 8]$ scl: 1



$[-2\pi, 2\pi]$ scl: $\frac{\pi}{2}$ by $[-4, 4]$ scl: 1