Chapter 7 Trigonometric Identities and Equations

7-1 Basic Trigonometric Identities

Page 427 Check for Understanding

1. Sample answer: $x = 45°$
2. Pythagorean identities are derived by applying the Pythagorean Theorem to a right triangle. The opposite angle identities are so named because $-A$ is the opposite of A.
3. $\tan\theta = \frac{1}{\cot\theta}$, $\cot\theta = \frac{1}{\tan\theta}$, $\frac{\cos\theta}{\sin\theta} = \cot\theta$,
$1 + \cot^2\theta = \csc^2\theta$
4. $\tan(-A) = \frac{\sin(-A)}{\cos(-A)}$
$= \frac{-\sin A}{\cos A}$
$= -\frac{\sin A}{\cos A}$
$= -\tan A$
5. Rosalinda is correct; there may be other values for which the equation is not true.
6. Sample answer: $\theta = 0°$
$\sin\theta + \cos\theta \stackrel{?}{=} \tan\theta$
$\sin 0° + \cos 0° \stackrel{?}{=} \tan 0°$
$0 + 1 \stackrel{?}{=} 0$
$1 \neq 0$
7. Sample answer: $x = 45°$
$\sec^2 x + \csc^2 x \stackrel{?}{=} 1$
$\sec^2 45° + \csc^2 45° \stackrel{?}{=} 1$
$(\sqrt{2})^2 + (\sqrt{2})^2 \stackrel{?}{=} 1$
$2 + 2 \stackrel{?}{=} 1$
$4 \neq 1$
8. $\sec\theta = \frac{1}{\cos\theta}$
$\sec\theta = \frac{1}{\frac{2}{3}}$
$\sec\theta = \frac{3}{2}$
9. $\tan\theta = \frac{1}{\cot\theta}$
$\tan\theta = \frac{1}{-\frac{\sqrt{5}}{2}}$
$\tan\theta = -\frac{2}{\sqrt{5}}$
$\tan\theta = -\frac{2\sqrt{5}}{5}$
10. $\sin^2\theta + \cos^2\theta = 1$
$\left(-\frac{1}{5}\right)^2 + \cos^2\theta = 1$
$\frac{1}{25} + \cos^2\theta = 1$
$\cos^2\theta = \frac{24}{25}$
$\cos\theta = \pm\frac{2\sqrt{6}}{5}$
Quadrant III, so $-\frac{2\sqrt{6}}{5}$
11. $\tan^2\theta + 1 = \sec^2\theta$
$\left(-\frac{4}{7}\right)^2 + 1 = \sec^2\theta$
$\frac{16}{49} + 1 = \sec^2\theta$
$\frac{65}{49} = \sec^2\theta$
$\pm\frac{\sqrt{65}}{7} = \sec\theta$
Quadrant IV, so $\frac{\sqrt{65}}{7}$
12. $\frac{7\pi}{3} = 2\pi + \frac{\pi}{3}$
$\cos\frac{7\pi}{3} = \cos\left(2\pi + \frac{\pi}{3}\right)$
$= \cos\frac{\pi}{3}$
13. $-330° = -360° + 30°$
$\csc(-330°) = \frac{1}{\sin(-330°)}$
$= \frac{1}{\sin(-360° + 30°)}$
$= \frac{1}{\sin 30°}$
$= \csc 30°$
14. $\frac{\csc\theta}{\cot\theta} = \frac{\frac{1}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}}$
$= \frac{1}{\sin\theta}\cdot\frac{\sin\theta}{\cos\theta}$
$= \frac{1}{\cos\theta}$
$= \sec\theta$
15. $\cos x \csc x \tan x = \cos x\left(\frac{1}{\sin x}\right)\left(\frac{\sin x}{\cos x}\right)$
$= 1$
16. $\cos x \cot x + \sin x = \cos x\left(\frac{\cos x}{\sin x}\right) + \sin x$
$= \frac{\cos^2 x}{\sin x} + \sin x$
$= \frac{\cos^2 x + \sin^2 x}{\sin x}$
$= \frac{1}{\sin x}$
$= \csc x$
17. $B = \frac{F\csc\theta}{I\ell}$
$BI\ell = F\csc\theta$
$F = \frac{BI\ell}{\csc\theta}$
$F = BI\ell\left(\frac{1}{\csc\theta}\right)$
$F = BI\ell\sin\theta$

Pages 427–430 Exercises

18. Sample answer: 45°
$\sin\theta\cos\theta \stackrel{?}{=} \cot\theta$
$\sin 45°\cos 45° \stackrel{?}{=} \cot 45°$
$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \stackrel{?}{=} 1$
$\frac{1}{2} \neq 1$
19. Sample answer: 45°
$\frac{\sec\theta}{\tan\theta} \stackrel{?}{=} \sin\theta$
$\frac{\sec 45°}{\tan 45°} \stackrel{?}{=} \sin 45°$
$\frac{\sqrt{2}}{1} \stackrel{?}{=} \frac{\sqrt{2}}{2}$
$\sqrt{2} \neq \frac{\sqrt{2}}{2}$

20. Sample answer: 30°

$$\sec^2 x - 1 \stackrel{?}{=} \frac{\cos x}{\csc x}$$
$$\sec^2 30° - 1 \stackrel{?}{=} \frac{\cos 30°}{\csc 30°}$$
$$\left(\frac{2\sqrt{3}}{3}\right)^2 - 1 \stackrel{?}{=} \frac{\frac{\sqrt{3}}{2}}{2}$$
$$\frac{12}{9} - 1 \stackrel{?}{=} \frac{\sqrt{3}}{4}$$
$$\frac{1}{3} \neq \frac{\sqrt{3}}{4}$$

21. Sample answer: 30°

$$\sin x + \cos x \stackrel{?}{=} 1$$
$$\sin 30° + \cos 30° \stackrel{?}{=} 1$$
$$\frac{1}{2} + \frac{\sqrt{3}}{2} \stackrel{?}{=} 1$$
$$\frac{1+\sqrt{3}}{2} \neq 1$$

22. Sample answer: 0°

$$\sin y \tan y \stackrel{?}{=} \cos y$$
$$\sin 0° \tan 0° \stackrel{?}{=} \cos 0°$$
$$0 \cdot 0 \stackrel{?}{=} 1$$
$$0 \neq 1$$

23. Sample answer: 45°

$$\tan^2 A + \cot^2 A \stackrel{?}{=} 1$$
$$\tan^2 45° + \cot^2 45° \stackrel{?}{=} 1$$
$$1 + 1 \stackrel{?}{=} 1$$
$$2 \neq 1$$

24. Sample answer: 0

$$\cos\left(\theta + \frac{\pi}{2}\right) \neq \cos\theta + \cos\frac{\pi}{2}$$
$$\cos\left(0 + \frac{\pi}{2}\right) \neq \cos 0 + \cos\frac{\pi}{2}$$
$$\cos\frac{\pi}{2} \neq \cos 0 + \cos\frac{\pi}{2}$$
$$0 \neq 1 + 0$$
$$0 \neq 1$$

25. $\csc\theta = \frac{1}{\sin\theta}$

$$\csc\theta = \frac{1}{\frac{2}{5}}$$
$$\csc\theta = \frac{5}{2}$$

26. $\cot\theta = \frac{1}{\tan\theta}$

$$\cot\theta = \frac{1}{\frac{\sqrt{3}}{4}}$$
$$\cot\theta = \frac{4}{\sqrt{3}}$$
$$\cot\theta = \frac{4\sqrt{3}}{3}$$

27. $\sin^2\theta + \cos^2\theta = 1$

$$\left(\frac{1}{4}\right)^2 + \cos^2\theta = 1$$
$$\frac{1}{16} + \cos^2\theta = 1$$
$$\cos^2\theta = \frac{15}{16}$$
$$\cos\theta = \pm\frac{\sqrt{15}}{4}$$

Quadrant I, so $\frac{\sqrt{15}}{4}$

28. $\sin^2\theta + \cos^2\theta = 1$

$$\sin^2\theta + \left(-\frac{2}{3}\right)^2 = 1$$
$$\sin^2\theta + \frac{4}{9} = 1$$
$$\sin^2\theta = \frac{5}{9}$$
$$\sin\theta = \pm\frac{\sqrt{5}}{3}$$

Quadrant II, so $\frac{\sqrt{5}}{3}$

29. $1 + \cot^2\theta = \csc^2\theta$

$$1 + \cot^2\theta = \left(\frac{\sqrt{11}}{3}\right)^2$$
$$1 + \cot^2\theta = \frac{11}{9}$$
$$\cot^2\theta = \frac{2}{9}$$
$$\cot\theta = \pm\frac{\sqrt{2}}{3}$$

Quadrant II, so $-\frac{\sqrt{2}}{3}$

30. $\tan^2\theta + 1 = \sec^2\theta$

$$\tan^2\theta + 1 = \left(-\frac{5}{4}\right)^2$$
$$\tan^2\theta + 1 = \frac{25}{16}$$
$$\tan^2\theta = \frac{9}{16}$$
$$\tan\theta = \pm\frac{3}{4}$$

Quadrant II, so $-\frac{3}{4}$

31. $\sin^2\theta + \cos^2\theta = 1$

$$\left(-\frac{1}{3}\right)^2 + \cos^2\theta = 1$$
$$\frac{1}{9} + \cos^2\theta = 1$$
$$\cos^2\theta = \frac{8}{9}$$
$$\cos\theta = \pm\frac{2\sqrt{2}}{3}$$

Quadrant III, so $\cos\theta = -\frac{2\sqrt{2}}{3}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\tan\theta = \frac{-\frac{1}{3}}{-\frac{2\sqrt{2}}{3}}$$
$$\tan\theta = \frac{1}{2\sqrt{2}} \quad \text{or} \quad \frac{\sqrt{2}}{4}$$

32. $\tan^2\theta + 1 = \sec^2\theta$

$$\left(\frac{2}{3}\right)^2 + 1 = \sec^2\theta$$
$$\frac{4}{9} + 1 = \sec^2\theta$$
$$\frac{13}{9} = \sec^2\theta$$
$$\pm\frac{\sqrt{13}}{3} = \sec\theta$$

Quadrant III, so $\sec\theta = -\frac{\sqrt{13}}{3}$

$$\cos\theta = \frac{1}{\sec\theta}$$
$$\cos\theta = \frac{1}{\frac{-\sqrt{13}}{3}}$$
$$\cos\theta = -\frac{3}{\sqrt{13}} \quad \text{or} \quad -\frac{3\sqrt{13}}{13}$$

33. $\cos\theta = \frac{1}{\sec\theta}$

$$\cos\theta = \frac{1}{-\frac{7}{5}}$$
$$\cos\theta = -\frac{5}{7}$$

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin^2\theta + \left(-\frac{5}{7}\right)^2 = 1$$
$$\sin^2\theta + \frac{25}{49} = 1$$
$$\sin^2\theta = \frac{24}{49}$$
$$\sin\theta = \pm\frac{2\sqrt{6}}{7}$$

Quadrant III, so $-\frac{2\sqrt{6}}{7}$

20. Sample answer: 30°

$\sec^2 x - 1 \stackrel{?}{=} \frac{\cos x}{\csc x}$

$\sec^2 30° - 1 \stackrel{?}{=} \frac{\cos 30°}{\csc 30°}$

$\left(\frac{2\sqrt{3}}{3}\right)^2 - 1 \stackrel{?}{=} \frac{\frac{\sqrt{3}}{2}}{2}$

$\frac{12}{9} - 1 \stackrel{?}{=} \frac{\sqrt{3}}{4}$

$\frac{1}{3} \neq \frac{\sqrt{3}}{4}$

21. Sample answer: 30°

$\sin x + \cos x \stackrel{?}{=} 1$

$\sin 30° + \cos 30° \stackrel{?}{=} 1$

$\frac{1}{2} + \frac{\sqrt{3}}{2} \stackrel{?}{=} 1$

$\frac{1+\sqrt{3}}{2} \neq 1$

22. Sample answer: 0°

$\sin y \tan y \stackrel{?}{=} \cos y$

$\sin 0° \tan 0° \stackrel{?}{=} \cos 0°$

$0 \cdot 0 \stackrel{?}{=} 1$

$0 \neq 1$

23. Sample answer: 45°

$\tan^2 A + \cot^2 A \stackrel{?}{=} 1$

$\tan^2 45° + \cot^2 45° \stackrel{?}{=} 1$

$1 + 1 \stackrel{?}{=} 1$

$2 \neq 1$

24. Sample answer: 0

$\cos\left(\theta + \frac{\pi}{2}\right) \neq \cos\theta + \cos\frac{\pi}{2}$

$\cos\left(0 + \frac{\pi}{2}\right) \neq \cos 0 + \cos\frac{\pi}{2}$

$\cos\frac{\pi}{2} \neq \cos 0 + \cos\frac{\pi}{2}$

$0 \neq 1 + 0$

$0 \neq 1$

25. $\csc\theta = \frac{1}{\sin\theta}$

$\csc\theta = \frac{1}{\frac{2}{5}}$

$\csc\theta = \frac{5}{2}$

26. $\cot\theta = \frac{1}{\tan\theta}$

$\cot\theta = \frac{1}{\frac{\sqrt{3}}{4}}$

$\cot\theta = \frac{4}{\sqrt{3}}$

$\cot\theta = \frac{4\sqrt{3}}{3}$

27. $\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{1}{4}\right)^2 + \cos^2\theta = 1$

$\frac{1}{16} + \cos^2\theta = 1$

$\cos^2\theta = \frac{15}{16}$

$\cos\theta = \pm\frac{\sqrt{15}}{4}$

Quadrant I, so $\frac{\sqrt{15}}{4}$

28. $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(-\frac{2}{3}\right)^2 = 1$

$\sin^2\theta + \frac{4}{9} = 1$

$\sin^2\theta = \frac{5}{9}$

$\sin\theta = \pm\frac{\sqrt{5}}{3}$

Quadrant II, so $\frac{\sqrt{5}}{3}$

29. $1 + \cot^2\theta = \csc^2\theta$

$1 + \cot^2\theta = \left(\frac{\sqrt{11}}{3}\right)^2$

$1 + \cot^2\theta = \frac{11}{9}$

$\cot^2\theta = \frac{2}{9}$

$\cot\theta = \pm\frac{\sqrt{2}}{3}$

Quadrant II, so $-\frac{\sqrt{2}}{3}$

30. $\tan^2\theta + 1 = \sec^2\theta$

$\tan^2\theta + 1 = \left(-\frac{5}{4}\right)^2$

$\tan^2\theta + 1 = \frac{25}{16}$

$\tan^2\theta = \frac{9}{16}$

$\tan\theta = \pm\frac{3}{4}$

Quadrant II, so $-\frac{3}{4}$

31. $\sin^2\theta + \cos^2\theta = 1$

$\left(-\frac{1}{3}\right)^2 + \cos^2\theta = 1$

$\frac{1}{9} + \cos^2\theta = 1$

$\cos^2\theta = \frac{8}{9}$

$\cos\theta = \pm\frac{2\sqrt{2}}{3}$

Quadrant III, so $\cos\theta = -\frac{2\sqrt{2}}{3}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\tan\theta = \frac{-\frac{1}{3}}{-\frac{2\sqrt{2}}{3}}$

$\tan\theta = \frac{1}{2\sqrt{2}}$ or $\frac{\sqrt{2}}{4}$

32. $\tan^2\theta + 1 = \sec^2\theta$

$\left(\frac{2}{3}\right)^2 + 1 = \sec^2\theta$

$\frac{4}{9} + 1 = \sec^2\theta$

$\frac{13}{9} = \sec^2\theta$

$\pm\frac{\sqrt{13}}{3} = \sec\theta$

Quadrant III, so $\sec\theta = -\frac{\sqrt{13}}{3}$

$\cos\theta = \frac{1}{\sec\theta}$

$\cos\theta = \frac{1}{-\frac{\sqrt{13}}{3}}$

$\cos\theta = -\frac{3}{\sqrt{13}}$ or $-\frac{3\sqrt{13}}{13}$

33. $\cos\theta = \frac{1}{\sec\theta}$

$\cos\theta = \frac{1}{-\frac{7}{5}}$

$\cos\theta = -\frac{5}{7}$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(-\frac{5}{7}\right)^2 = 1$

$\sin^2\theta + \frac{25}{49} = 1$

$\sin^2\theta = \frac{24}{49}$

$\sin\theta = \pm\frac{2\sqrt{6}}{7}$

Quadrant III, so $-\frac{2\sqrt{6}}{7}$

34. $\sec\theta = \frac{1}{\cos\theta}$

$\sec\theta = \frac{1}{\frac{1}{8}}$

$\sec\theta = 8$

$\tan^2\theta + 1 = \sec^2\theta$

$\tan^2\theta + 1 = 8^2$

$\tan^2\theta + 1 = 64$

$\tan^2\theta = 63$

$\tan\theta = \pm 3\sqrt{7}$

Quadrant IV, so $-3\sqrt{7}$

35. $1 + \cot^2\theta = \csc^2\theta$

$1 + \left(-\frac{4}{3}\right)^2 = \csc^2\theta$

$1 + \frac{16}{9} = \csc^2\theta$

$\frac{25}{9} = \csc^2\theta$

$\pm\frac{5}{3} = \csc\theta$

Quadrant IV, so $-\frac{5}{3}$

$\sin\theta = \frac{1}{\csc\theta}$

$\sin\theta = \frac{1}{-\frac{5}{3}}$

$\sin\theta = -\frac{3}{5}$

36. $1 + \cot^2\theta = \csc^2\theta$

$1 + (-8)^2 = \csc^2\theta$

$1 + 64 = \csc^2\theta$

$65 = \csc^2\theta$

$\pm\sqrt{65} = \csc\theta$

Quadrant IV, so $-\sqrt{65}$

37. $\sec\theta = \frac{1}{\cos\theta}$

$\sec\theta = \frac{1}{-\frac{\sqrt{3}}{4}}$

$\sec\theta = -\frac{4}{\sqrt{3}}$ or $-\frac{4\sqrt{3}}{3}$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(-\frac{\sqrt{3}}{4}\right)^2 = 1$

$\sin^2\theta + \frac{3}{16} = 1$

$\sin^2\theta = \frac{13}{16}$

$\sin\theta = \pm\frac{\sqrt{13}}{4}$

Quadrant II, so $\frac{\sqrt{13}}{4}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\tan\theta = \frac{\frac{\sqrt{13}}{4}}{-\frac{\sqrt{3}}{4}}$

$\tan\theta = -\frac{\sqrt{13}}{\sqrt{3}}$ or $-\frac{\sqrt{39}}{3}$

$\frac{\sec^2 A - \tan^2 A}{2\sin^2 A + 2\cos^2 A} = \frac{\left(-\frac{4\sqrt{3}}{3}\right)^2 - \left(-\frac{\sqrt{39}}{3}\right)^2}{2\left(\frac{\sqrt{13}}{4}\right)^2 + 2\left(-\frac{\sqrt{3}}{4}\right)^2}$

$= \frac{\frac{48}{9} - \frac{39}{9}}{2\left(\frac{13}{16}\right) + 2\left(\frac{3}{16}\right)}$

$= \frac{\frac{9}{9}}{\frac{32}{16}}$

$= \frac{1}{2}$

38. $390° = 360° + 30°$

$\sin 390° = \sin(360° + 30°)$

$= \sin 30°$

39. $\frac{27\pi}{8} = 3\pi + \frac{3\pi}{8}$

$\cos\frac{27\pi}{8} = \cos\left(3\pi + \frac{3\pi}{8}\right)$

$= -\cos\frac{3\pi}{8}$

40. $\frac{19\pi}{5} = 2(2\pi) - \frac{\pi}{5}$

$\tan\frac{19\pi}{5} = \frac{\sin\frac{19\pi}{5}}{\cos\frac{19\pi}{5}}$

$= \frac{\sin\left(2(2\pi) - \frac{\pi}{5}\right)}{\cos\left(2(2\pi) - \frac{\pi}{5}\right)}$

$= \frac{-\sin\frac{\pi}{5}}{\cos\frac{\pi}{5}}$

$= -\tan\frac{\pi}{5}$

41. $\frac{10\pi}{3} = 3\pi + \frac{\pi}{3}$

$\csc\frac{10\pi}{3} = \frac{1}{\sin\frac{10\pi}{3}}$

$= \frac{1}{\sin\left(3\pi + \frac{\pi}{3}\right)}$

$= \frac{1}{-\sin\frac{\pi}{3}}$

$= -\csc\frac{\pi}{3}$

42. $-1290° = -7(180°) - 30°$

$\sec(-1290°) = \frac{1}{\cos(-1290°)}$

$= \frac{1}{\cos(-7(180°) - 30°)}$

$= \frac{1}{-\cos 30°}$

$= -\sec 30°$

43. $-660° = -2(360°) + 60°$

$\cot(-660°) = \frac{\cos(-660°)}{\sin(-660°)}$

$= \frac{\cos(-2(360°) + 60°)}{\sin(-2(360°) + 60°)}$

$= \frac{\cos 60°}{\sin 60°}$

$= \cot 60°$

44. $\frac{\sec x}{\tan x} = \frac{\frac{1}{\cos x}}{\frac{\sin x}{\cos x}}$

$= \frac{1}{\sin x}$

$= \csc x$

45. $\frac{\cot\theta}{\cos\theta} = \frac{\frac{\cos\theta}{\sin\theta}}{\cos\theta}$

$= \frac{1}{\sin\theta}$

$= \csc\theta$

46. $\frac{\sin(\theta + \pi)}{\cos(\theta - \pi)} = \frac{-\sin\theta}{-\cos\theta}$

$= \tan\theta$

47. $(\sin x + \cos x)^2 + (\sin x - \cos x)^2$

$= \sin^2 x + 2\sin x\cos x + \cos^2 x + \sin^2 x$

$- 2\sin x\cos x + \cos^2 x$

$= 2\sin^2 x + 2\cos^2 x$

$= 2(\sin^2 x + \cos^2 x)$

$= 2$

48. $\sin x \cos x \sec x \cot x = \sin x \cos x\left(\frac{1}{\cos x}\right)\left(\frac{\cos x}{\sin x}\right)$
$= \cos x$

49. $\cos x \tan x + \sin x \cot x = \cos x\left(\frac{\sin x}{\cos x}\right) + \sin x\left(\frac{\cos x}{\sin x}\right)$
$= \sin x + \cos x$

50. $(1 + \cos\theta)(\csc\theta - \cot\theta) = (1 + \cos\theta)\left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)$
$= (1 + \cos\theta)\left(\frac{1 - \cos\theta}{\sin\theta}\right)$
$= \frac{1 - \cos^2\theta}{\sin\theta}$
$= \frac{\sin^2\theta}{\sin\theta}$
$= \sin\theta$

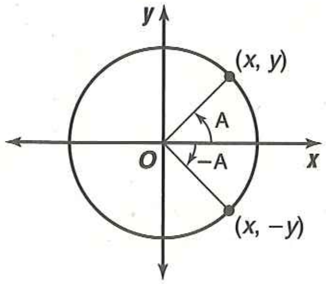
51. $1 + \cot^2\theta - \cos^2\theta - \cos^2\theta\cot^2\theta$
$= 1 + \cot^2\theta - \cos^2\theta(1 + \cot^2\theta)$
$= \csc^2\theta - \cos^2\theta(\csc^2\theta)$
$= \csc^2\theta(1 - \cos^2\theta)$
$= \csc^2\theta(\sin^2\theta)$
$= \frac{1}{\sin^2\theta}(\sin^2\theta)$
$= 1$

52. $\frac{\sin x}{1 + \cos x} + \frac{\sin x}{1 - \cos x}$
$= \frac{\sin x - \sin x\cos x}{1 - \cos^2 x} + \frac{\sin x + \sin x\cos x}{1 - \cos^2 x}$
$= \frac{2\sin x}{1 - \cos^2 x}$
$= \frac{2\sin x}{\sin^2 x}$
$= \frac{2}{\sin x}$
$= 2\csc x$

53. $\cos^4\alpha + 2\cos^2\alpha\sin^2\alpha + \sin^4\alpha = (\cos^2\alpha + \sin^2\alpha)^2$
$= 1^2 \text{ or } 1$

54.
$I = I_0\cos^2\theta$
$0 = I_0\cos^2\theta$
$0 = \cos^2\theta$
$0 = \cos\theta$
$\cos^{-1}0 = \theta$
$90° = \theta$

55. Let (x, y) be the point where the terminal side of A intersects the unit circle when A is in standard position. When A is reflected about the x-axis to obtain $-A$, the y-coordinate is multiplied by -1, but the x-coordinate is unchanged. So,
$\sin(-A) = -y = -\sin A$ and
$\cos(-A) = x = \cos A$.

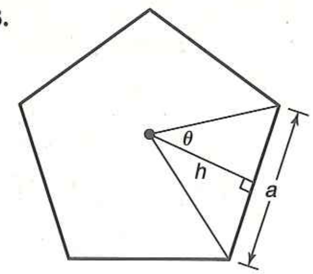


56a. $e = \frac{W\sec\theta}{As}$
$eAs = W\sec\theta$
$\frac{eAs}{\sec\theta} = W$
$W = eAs\cos\theta$

56b. $W = eAs\cos\theta$
$W = 0.80(0.75)(1000)\cos 40°$
$W \approx 459.6266659$
459.63 W

57. $F_N - mg\cos\theta = 0$
$F_N = mg\cos\theta$
$mg\sin\theta - \mu_k F_N = 0$
$mg\sin\theta - \mu_k(mg\cos\theta) = 0$
$\mu_k(mg\cos\theta) = mg\sin\theta$
$\mu_k = \frac{mg\sin\theta}{mg\cos\theta}$
$\mu_k = \frac{\sin\theta}{\cos\theta}$
$\mu_k = \tan\theta$

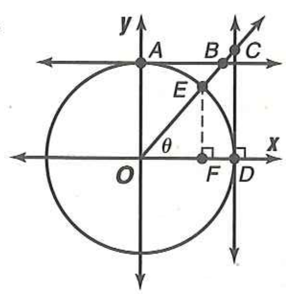
58.



$\theta = \frac{360°}{2n} = \frac{180°}{n}$, $\tan\theta = \frac{\frac{a}{2}}{h}$, so $h = \frac{a}{2\tan\theta} = \frac{a}{2}\cot\theta$.

The area of the isosceles triangle is $\frac{1}{2}(a)\left(\frac{a}{2}\cot\frac{180°}{n}\right)$ $= \frac{a^2}{4}\cot\left(\frac{180°}{n}\right)$. There are n such triangles, so $A = \frac{1}{4}na^2\cot\left(\frac{180°}{n}\right)$.

59.



$\sin\theta = EF$ and $\cos\theta = OF$ since the circle is a unit circle. $\tan\theta = \frac{CD}{OD} = \frac{CD}{1} = CD$.
$\sec\theta = \frac{CO}{OD} = \frac{CO}{1} = CO$. $\triangle EOF \sim \triangle OBA$, so $\frac{OF}{EF} = \frac{BA}{OA} = \frac{BA}{1} = BA$. Then $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{OF}{EF} = BA$.
Also by similar triangles, $\frac{EO}{EF} = \frac{OB}{OA}$, or $\frac{1}{EF} = \frac{OB}{1}$.
Then $\csc\theta = \frac{1}{\sin\theta} = \frac{1}{EF} = \frac{OB}{1} = OB$.

60. $\text{Cos}^{-1}\left(-\frac{\sqrt{2}}{2}\right) = 135°$