

# Chapter 7 Trigonometric Identities and Equations

## 7-1 Basic Trigonometric Identities

### Page 427 Check for Understanding

1. Sample answer:  $x = 45^\circ$

2. Pythagorean identities are derived by applying the Pythagorean Theorem to a right triangle. The opposite angle identities are so named because  $-A$  is the opposite of  $A$ .

3.  $\tan \theta = \frac{1}{\cot \theta}$ ,  $\cot \theta = \frac{1}{\tan \theta}$ ,  $\frac{\cos \theta}{\sin \theta} = \cot \theta$ ,

$$1 + \cot^2 \theta = \csc^2 \theta$$

4.  $\tan(-A) = \frac{\sin(-A)}{\cos(-A)}$

$$= \frac{-\sin A}{\cos A}$$

$$= -\frac{\sin A}{\cos A}$$

$$= -\tan A$$

5. Rosalinda is correct; there may be other values for which the equation is not true.

6. Sample answer:  $\theta = 0^\circ$

$$\begin{aligned}\sin \theta + \cos \theta &\stackrel{?}{=} \tan \theta \\ \sin 0^\circ + \cos 0^\circ &\stackrel{?}{=} \tan 0^\circ \\ 0 + 1 &\stackrel{?}{=} 0 \\ 1 &\neq 0\end{aligned}$$

7. Sample answer:  $x = 45^\circ$

$$\begin{aligned}\sec^2 x + \csc^2 x &\stackrel{?}{=} 1 \\ \sec^2 45^\circ + \csc^2 45^\circ &\stackrel{?}{=} 1 \\ (\sqrt{2})^2 + (\sqrt{2})^2 &\stackrel{?}{=} 1 \\ 2 + 2 &\stackrel{?}{=} 1 \\ 4 &\neq 1\end{aligned}$$

8.  $\sec \theta = \frac{1}{\cos \theta}$

$$\sec \theta = \frac{1}{\frac{2}{3}}$$

$$\sec \theta = \frac{3}{2}$$

9.  $\tan \theta = \frac{1}{\cot \theta}$

$$\tan \theta = \frac{1}{-\frac{\sqrt{5}}{2}}$$

$$\tan \theta = -\frac{2}{\sqrt{5}}$$

$$\tan \theta = -\frac{2\sqrt{5}}{5}$$

10.  $\sin^2 \theta + \cos^2 \theta = 1$

$$\begin{aligned}\left(-\frac{1}{5}\right)^2 + \cos^2 \theta &= 1 \\ \frac{1}{25} + \cos^2 \theta &= 1 \\ \cos^2 \theta &= \frac{24}{25} \\ \cos \theta &= \pm \frac{2\sqrt{6}}{5}\end{aligned}$$

Quadrant III, so  $-\frac{2\sqrt{6}}{5}$

11.  $\tan^2 \theta + 1 = \sec^2 \theta$

$$\begin{aligned}\left(-\frac{4}{7}\right)^2 + 1 &= \sec^2 \theta \\ \frac{16}{49} + 1 &= \sec^2 \theta \\ \frac{65}{49} &= \sec^2 \theta \\ \pm \frac{\sqrt{65}}{7} &= \sec \theta\end{aligned}$$

Quadrant IV, so  $\frac{\sqrt{65}}{7}$

12.  $\frac{7\pi}{3} = 2\pi + \frac{\pi}{3}$

$$\begin{aligned}\cos \frac{7\pi}{3} &= \cos\left(2\pi + \frac{\pi}{3}\right) \\ &= \cos \frac{\pi}{3}\end{aligned}$$

13.  $-330^\circ = -360^\circ + 30^\circ$

$$\begin{aligned}\csc(-330^\circ) &= \frac{1}{\sin(-330^\circ)} \\ &= \frac{1}{\sin(-360^\circ + 30^\circ)} \\ &= \frac{1}{\sin 30^\circ} \\ &= \csc 30^\circ\end{aligned}$$

14.  $\frac{\csc \theta}{\cot \theta} = \frac{\frac{\sin \theta}{\cos \theta}}{\frac{\sin \theta}{\cos \theta}}$

$$\begin{aligned}&= \frac{1}{\sin \theta} \cdot \frac{\sin \theta}{\cos \theta} \\ &= \frac{1}{\cos \theta} \\ &= \sec \theta\end{aligned}$$

15.  $\cos x \csc x \tan x = \cos x \left(\frac{1}{\sin x}\right) \left(\frac{\sin x}{\cos x}\right)$

$$= 1$$

16.  $\cos x \cot x + \sin x = \cos x \left(\frac{\cos x}{\sin x}\right) + \sin x$

$$\begin{aligned}&= \frac{\cos^2 x}{\sin x} + \sin x \\ &= \frac{\cos^2 x + \sin^2 x}{\sin x} \\ &= \frac{1}{\sin x} \\ &= \csc x\end{aligned}$$

17.  $B = \frac{F \csc \theta}{I\ell}$

$$BI\ell = F \csc \theta$$

$$F = \frac{BI\ell}{\csc \theta}$$

$$F = BI\ell \left(\frac{1}{\csc \theta}\right)$$

$$F = BI\ell \sin \theta$$

### Pages 427–430 Exercises

18. Sample answer:  $45^\circ$

$$\sin \theta \cos \theta \stackrel{?}{=} \cot \theta$$

$$\sin 45^\circ \cos 45^\circ \stackrel{?}{=} \cot 45^\circ$$

$$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \stackrel{?}{=} 1$$

$$\frac{1}{2} \neq 1$$

19. Sample answer:  $45^\circ$

$$\frac{\sec \theta}{\tan \theta} \stackrel{?}{=} \sin \theta$$

$$\frac{\sec 45^\circ}{\tan 45^\circ} \stackrel{?}{=} \sin 45^\circ$$

$$\frac{\sqrt{2}}{1} \stackrel{?}{=} \frac{\sqrt{2}}{2}$$

$$\sqrt{2} \neq \frac{\sqrt{2}}{2}$$

20. Sample answer:  $30^\circ$

$$\begin{aligned}\sec^2 x - 1 &\stackrel{?}{=} \frac{\cos x}{\csc x} \\ \sec^2 30^\circ - 1 &\stackrel{?}{=} \frac{\cos 30^\circ}{\csc 30^\circ} \\ \left(\frac{2\sqrt{3}}{3}\right)^2 - 1 &\stackrel{?}{=} \frac{\frac{\sqrt{3}}{2}}{2} \\ \frac{12}{9} - 1 &\stackrel{?}{=} \frac{\frac{\sqrt{3}}{4}}{2} \\ \frac{1}{3} &\neq \frac{\sqrt{3}}{4}\end{aligned}$$

21. Sample answer:  $30^\circ$

$$\begin{aligned}\sin x + \cos x &\stackrel{?}{=} 1 \\ \sin 30^\circ + \cos 30^\circ &\stackrel{?}{=} 1 \\ \frac{1}{2} + \frac{\sqrt{3}}{2} &\stackrel{?}{=} 1 \\ \frac{1+\sqrt{3}}{2} &\neq 1\end{aligned}$$

22. Sample answer:  $0^\circ$

$$\begin{aligned}\sin y \tan y &\stackrel{?}{=} \cos y \\ \sin 0^\circ \tan 0^\circ &\stackrel{?}{=} \cos 0^\circ \\ 0 \cdot 0 &\stackrel{?}{=} 1 \\ 0 &\neq 1\end{aligned}$$

23. Sample answer:  $45^\circ$

$$\begin{aligned}\tan^2 A + \cot^2 A &\stackrel{?}{=} 1 \\ \tan^2 45^\circ + \cot^2 45^\circ &\stackrel{?}{=} 1 \\ 1 + 1 &\stackrel{?}{=} 1 \\ 2 &\neq 1\end{aligned}$$

24. Sample answer: 0

$$\begin{aligned}\cos\left(\theta + \frac{\pi}{2}\right) &\neq \cos\theta + \cos\frac{\pi}{2} \\ \cos\left(0 + \frac{\pi}{2}\right) &\neq \cos 0 + \cos\frac{\pi}{2} \\ \cos\frac{\pi}{2} &\neq \cos 0 + \cos\frac{\pi}{2} \\ 0 &\neq 1 + 0 \\ 0 &\neq 1\end{aligned}$$

25.  $\csc\theta = \frac{1}{\sin\theta}$

$$\csc\theta = \frac{1}{\frac{2}{5}}$$

$$\csc\theta = \frac{5}{2}$$

26.  $\cot\theta = \frac{1}{\tan\theta}$

$$\cot\theta = \frac{1}{\frac{\sqrt{3}}{4}}$$

$$\cot\theta = \frac{4}{\sqrt{3}}$$

$$\cot\theta = \frac{4\sqrt{3}}{3}$$

27.  $\sin^2\theta + \cos^2\theta = 1$

$$\left(\frac{1}{4}\right)^2 + \cos^2\theta = 1$$

$$\frac{1}{16} + \cos^2\theta = 1$$

$$\cos^2\theta = \frac{15}{16}$$

$$\cos\theta = \pm\frac{\sqrt{15}}{4}$$

Quadrant I, so  $\frac{\sqrt{15}}{4}$

28.  $\sin^2\theta + \cos^2\theta = 1$

$$\sin^2\theta + \left(-\frac{2}{3}\right)^2 = 1$$

$$\sin^2\theta + \frac{4}{9} = 1$$

$$\sin^2\theta = \frac{5}{9}$$

$$\sin\theta = \pm\frac{\sqrt{5}}{3}$$

Quadrant II, so  $\frac{\sqrt{5}}{3}$

29.  $1 + \cot^2\theta = \csc^2\theta$

$$1 + \cot^2\theta = \left(\frac{\sqrt{11}}{3}\right)^2$$

$$1 + \cot^2\theta = \frac{11}{9}$$

$$\cot^2\theta = \frac{2}{9}$$

$$\cot\theta = \pm\frac{\sqrt{2}}{3}$$

Quadrant II, so  $-\frac{\sqrt{2}}{3}$

30.  $\tan^2\theta + 1 = \sec^2\theta$

$$\tan^2\theta + 1 = \left(-\frac{5}{4}\right)^2$$

$$\tan^2\theta + 1 = \frac{25}{16}$$

$$\tan^2\theta = \frac{9}{16}$$

$$\tan\theta = \pm\frac{3}{4}$$

Quadrant II, so  $-\frac{3}{4}$

31.  $\sin^2\theta + \cos^2\theta = 1$

$$\left(-\frac{1}{3}\right)^2 + \cos^2\theta = 1$$

$$\frac{1}{9} + \cos^2\theta = 1$$

$$\cos^2\theta = \frac{8}{9}$$

$$\cos\theta = \pm\frac{2\sqrt{2}}{3}$$

Quadrant III, so  $\cos\theta = -\frac{2\sqrt{2}}{3}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\tan\theta = \frac{-\frac{1}{3}}{-\frac{2\sqrt{2}}{3}}$$

$$\tan\theta = \frac{1}{2\sqrt{2}} \quad \text{or} \quad \frac{\sqrt{2}}{4}$$

32.  $\tan^2\theta + 1 = \sec^2\theta$

$$\left(\frac{2}{3}\right)^2 + 1 = \sec^2\theta$$

$$\frac{4}{9} + 1 = \sec^2\theta$$

$$\frac{13}{9} = \sec^2\theta$$

$$\pm\frac{\sqrt{13}}{3} = \sec\theta$$

Quadrant III, so  $\sec\theta = -\frac{\sqrt{13}}{3}$

$$\cos\theta = \frac{1}{\sec\theta}$$

$$\cos\theta = \frac{1}{-\frac{\sqrt{13}}{3}}$$

$$\cos\theta = -\frac{3}{\sqrt{13}} \quad \text{or} \quad -\frac{3\sqrt{13}}{13}$$

33.  $\cos\theta = \frac{1}{\sec\theta}$

$$\cos\theta = \frac{1}{-\frac{7}{5}}$$

$$\cos\theta = -\frac{5}{7}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta + \left(-\frac{5}{7}\right)^2 = 1$$

$$\sin^2\theta + \frac{25}{49} = 1$$

$$\sin^2\theta = \frac{24}{49}$$

$$\sin\theta = \pm\frac{2\sqrt{6}}{7}$$

Quadrant III, so  $-\frac{2\sqrt{6}}{7}$

20. Sample answer:  $30^\circ$

$$\begin{aligned}\sec^2 x - 1 &\stackrel{?}{=} \frac{\cos x}{\csc x} \\ \sec^2 30^\circ - 1 &\stackrel{?}{=} \frac{\cos 30^\circ}{\csc 30^\circ} \\ \left(\frac{2\sqrt{3}}{3}\right)^2 - 1 &\stackrel{?}{=} \frac{\frac{\sqrt{3}}{2}}{2} \\ \frac{12}{9} - 1 &\stackrel{?}{=} \frac{\sqrt{3}}{4} \\ \frac{1}{3} &\neq \frac{\sqrt{3}}{4}\end{aligned}$$

21. Sample answer:  $30^\circ$

$$\begin{aligned}\sin x + \cos x &\stackrel{?}{=} 1 \\ \sin 30^\circ + \cos 30^\circ &\stackrel{?}{=} 1 \\ \frac{1}{2} + \frac{\sqrt{3}}{2} &\stackrel{?}{=} 1 \\ \frac{1+\sqrt{3}}{2} &\neq 1\end{aligned}$$

22. Sample answer:  $0^\circ$

$$\begin{aligned}\sin y \tan y &\stackrel{?}{=} \cos y \\ \sin 0^\circ \tan 0^\circ &\stackrel{?}{=} \cos 0^\circ \\ 0 \cdot 0 &\stackrel{?}{=} 1 \\ 0 &\neq 1\end{aligned}$$

23. Sample answer:  $45^\circ$

$$\begin{aligned}\tan^2 A + \cot^2 A &\stackrel{?}{=} 1 \\ \tan^2 45^\circ + \cot^2 45^\circ &\stackrel{?}{=} 1 \\ 1 + 1 &\stackrel{?}{=} 1 \\ 2 &\neq 1\end{aligned}$$

24. Sample answer:  $0$

$$\begin{aligned}\cos\left(\theta + \frac{\pi}{2}\right) &\neq \cos\theta + \cos\frac{\pi}{2} \\ \cos\left(0 + \frac{\pi}{2}\right) &\neq \cos 0 + \cos\frac{\pi}{2} \\ \cos\frac{\pi}{2} &\neq \cos 0 + \cos\frac{\pi}{2} \\ 0 &\neq 1 + 0 \\ 0 &\neq 1\end{aligned}$$

25.  $\csc\theta = \frac{1}{\sin\theta}$

$$\begin{aligned}\csc\theta &= \frac{1}{2} \\ &\quad \text{or} \\ \csc\theta &= \frac{5}{2} \\ &\quad \text{or} \\ \csc\theta &= \frac{4\sqrt{3}}{3}\end{aligned}$$

27.  $\sin^2\theta + \cos^2\theta = 1$

$$\begin{aligned}\left(\frac{1}{4}\right)^2 + \cos^2\theta &= 1 \\ \frac{1}{16} + \cos^2\theta &= 1 \\ \cos^2\theta &= \frac{15}{16} \\ \cos\theta &= \pm\frac{\sqrt{15}}{4}\end{aligned}$$

Quadrant I, so  $\frac{\sqrt{15}}{4}$

28.  $\sin^2\theta + \cos^2\theta = 1$

$$\begin{aligned}\sin^2\theta + \left(-\frac{2}{3}\right)^2 &= 1 \\ \sin^2\theta + \frac{4}{9} &= 1 \\ \sin^2\theta &= \frac{5}{9} \\ \sin\theta &= \pm\frac{\sqrt{5}}{3}\end{aligned}$$

Quadrant II, so  $\frac{\sqrt{5}}{3}$

29.  $1 + \cot^2\theta = \csc^2\theta$

$$\begin{aligned}1 + \cot^2\theta &= \left(\frac{\sqrt{11}}{3}\right)^2 \\ 1 + \cot^2\theta &= \frac{11}{9} \\ \cot^2\theta &= \frac{2}{9} \\ \cot\theta &= \pm\frac{\sqrt{2}}{3}\end{aligned}$$

Quadrant II, so  $-\frac{\sqrt{2}}{3}$

30.  $\tan^2\theta + 1 = \sec^2\theta$

$$\begin{aligned}\tan^2\theta + 1 &= \left(-\frac{5}{4}\right)^2 \\ \tan^2\theta + 1 &= \frac{25}{16} \\ \tan^2\theta &= \frac{9}{16} \\ \tan\theta &= \pm\frac{3}{4}\end{aligned}$$

Quadrant II, so  $-\frac{3}{4}$

31.  $\sin^2\theta + \cos^2\theta = 1$

$$\begin{aligned}\left(-\frac{1}{3}\right)^2 + \cos^2\theta &= 1 \\ \frac{1}{9} + \cos^2\theta &= 1 \\ \cos^2\theta &= \frac{8}{9} \\ \cos\theta &= \pm\frac{2\sqrt{2}}{3}\end{aligned}$$

Quadrant III, so  $\cos\theta = -\frac{2\sqrt{2}}{3}$

$$\begin{aligned}\tan\theta &= \frac{\sin\theta}{\cos\theta} \\ \tan\theta &= \frac{-\frac{1}{3}}{-\frac{2\sqrt{2}}{3}} \\ \tan\theta &= \frac{1}{2\sqrt{2}} \quad \text{or} \quad \frac{\sqrt{2}}{4}\end{aligned}$$

32.  $\tan^2\theta + 1 = \sec^2\theta$

$$\begin{aligned}\left(\frac{2}{3}\right)^2 + 1 &= \sec^2\theta \\ \frac{4}{9} + 1 &= \sec^2\theta \\ \frac{13}{9} &= \sec^2\theta \\ \pm\frac{\sqrt{13}}{3} &= \sec\theta\end{aligned}$$

Quadrant III, so  $\sec\theta = -\frac{\sqrt{13}}{3}$

$$\begin{aligned}\cos\theta &= \frac{1}{\sec\theta} \\ \cos\theta &= \frac{1}{-\frac{\sqrt{13}}{3}} \\ \cos\theta &= -\frac{3}{\sqrt{13}} \quad \text{or} \quad -\frac{3\sqrt{13}}{13}\end{aligned}$$

33.  $\cos\theta = \frac{1}{\sec\theta}$

$$\begin{aligned}\cos\theta &= \frac{1}{-\frac{7}{5}} \\ \cos\theta &= -\frac{5}{7} \\ \sin^2\theta + \cos^2\theta &= 1 \\ \sin^2\theta + \left(-\frac{5}{7}\right)^2 &= 1 \\ \sin^2\theta + \frac{25}{49} &= 1 \\ \sin^2\theta &= \frac{24}{49} \\ \sin\theta &= \pm\frac{2\sqrt{6}}{7}\end{aligned}$$

Quadrant III, so  $-\frac{2\sqrt{6}}{7}$

34.  $\sec\theta = \frac{1}{\cos\theta}$

$$\begin{aligned}\sec\theta &= \frac{1}{\frac{1}{8}} \\ \sec\theta &= 8 \\ \tan^2\theta + 1 &= \sec^2\theta \\ \tan^2\theta + 1 &= 64 \\ \tan^2\theta &= 63 \\ \tan\theta &= \pm 3\sqrt{7}\end{aligned}$$

Quadrant IV, so  $-3\sqrt{7}$

35.  $1 + \cot^2\theta = \csc^2\theta$

$$\begin{aligned}1 + \left(-\frac{4}{3}\right)^2 &= \csc^2\theta \\ 1 + \frac{16}{9} &= \csc^2\theta \\ \frac{25}{9} &= \csc^2\theta \\ \pm\frac{5}{3} &= \csc\theta\end{aligned}$$

Quadrant IV, so  $-\frac{5}{3}$

36.  $1 + \cot^2\theta = \csc^2\theta$

$$\begin{aligned}1 + (-8)^2 &= \csc^2\theta \\ 1 + 64 &= \csc^2\theta \\ 65 &= \csc^2\theta \\ \pm\sqrt{65} &= \csc\theta\end{aligned}$$

Quadrant IV, so  $-\sqrt{65}$

37.  $\sec\theta = \frac{1}{\cos\theta}$

$$\begin{aligned}\sec\theta &= \frac{1}{-\frac{\sqrt{3}}{4}} \\ \sec\theta &= -\frac{4}{\sqrt{3}} \quad \text{or} \quad -\frac{4\sqrt{3}}{3} \\ \sin^2\theta + \cos^2\theta &= 1 \\ \sin^2\theta + \left(-\frac{\sqrt{3}}{4}\right)^2 &= 1 \\ \sin^2\theta + \frac{3}{16} &= 1 \\ \sin^2\theta &= \frac{13}{16} \\ \sin\theta &= \pm\frac{\sqrt{13}}{4}\end{aligned}$$

Quadrant II, so  $\frac{\sqrt{13}}{4}$

38.  $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$$\begin{aligned}\tan\theta &= \frac{\sqrt{13}}{4} \\ \tan\theta &= -\frac{\sqrt{13}}{3} \quad \text{or} \quad -\frac{\sqrt{39}}{3} \\ \frac{\sec^2\theta - \tan^2\theta}{2\sin^2\theta + 2\cos^2\theta} &= \frac{\left(\frac{4\sqrt{3}}{3}\right)^2 - \left(\frac{\sqrt{39}}{3}\right)^2}{2\left(\frac{\sqrt{13}}{4}\right)^2 + 2\left(-\frac{\sqrt{3}}{4}\right)^2} \\ &= \frac{\frac{48}{9} - \frac{39}{9}}{2\left(\frac{13}{16}\right) + 2\left(\frac{3}{16}\right)} \\ &= \frac{\frac{9}{9}}{\frac{32}{16}} \\ &= \frac{1}{2}\end{aligned}$$

38.  $390^\circ = 360^\circ + 30^\circ$

$\sin 390^\circ = \sin(360^\circ + 30^\circ)$

$= \sin 30^\circ$

39.  $\frac{27\pi}{8} = 3\pi + \frac{3\pi}{8}$

$$\begin{aligned}\cos\frac{27\pi}{8} &= \cos\left(3\pi + \frac{3\pi}{8}\right) \\ &= -\cos\frac{3\pi}{8}\end{aligned}$$

40.  $\frac{19\pi}{5} = 2(2\pi) - \frac{\pi}{5}$

$$\begin{aligned}\tan\frac{19\pi}{5} &= \frac{\sin\frac{19\pi}{5}}{\cos\frac{19\pi}{5}} \\ &= \frac{\sin(2(2\pi) - \frac{\pi}{5})}{\cos(2(2\pi) - \frac{\pi}{5})} \\ &= \frac{-\sin\frac{\pi}{5}}{\cos\frac{\pi}{5}} \\ &= -\tan\frac{\pi}{5}\end{aligned}$$

41.  $\frac{10\pi}{3} = 3\pi + \frac{\pi}{3}$

$$\begin{aligned}\csc\frac{10\pi}{3} &= \frac{1}{\sin\frac{10\pi}{3}} \\ &= \frac{1}{\sin(3\pi + \frac{\pi}{3})} \\ &= \frac{1}{-\sin\frac{\pi}{3}} \\ &= -\csc\frac{\pi}{3}\end{aligned}$$

42.  $-1290^\circ = -7(180^\circ) - 30^\circ$

$$\begin{aligned}\sec(-1290^\circ) &= \frac{1}{\cos(-1290^\circ)} \\ &= \frac{1}{\cos(-7(180^\circ) - 30^\circ)} \\ &= \frac{1}{-\cos 30^\circ} \\ &= -\sec 30^\circ\end{aligned}$$

43.  $-660^\circ = -2(360^\circ) + 60^\circ$

$$\begin{aligned}\cot(-660^\circ) &= \frac{\cos(-660^\circ)}{\sin(-660^\circ)} \\ &= \frac{\cos(-2(360^\circ) + 60^\circ)}{\sin(-2(360^\circ) + 60^\circ)} \\ &= \frac{\cos 60^\circ}{\sin 60^\circ} \\ &= \cot 60^\circ\end{aligned}$$

44.  $\frac{\sec x}{\tan x} = \frac{\frac{1}{\cos x}}{\frac{\sin x}{\cos x}}$

$$\begin{aligned}&= \frac{1}{\sin x} \\ &= \csc x\end{aligned}$$

45.  $\frac{\cot\theta}{\cos\theta} = \frac{\frac{\cos\theta}{\sin\theta}}{\cos\theta}$

$$\begin{aligned}&= \frac{1}{\sin\theta} \\ &= \csc\theta\end{aligned}$$

46.  $\frac{\sin(\theta + \pi)}{\cos(\theta - \pi)} = \frac{-\sin\theta}{-\cos\theta}$

47.  $(\sin x + \cos x)^2 + (\sin x - \cos x)^2$

$$\begin{aligned}&= \sin^2 x + 2\sin x \cos x + \cos^2 x + \sin^2 x \\ &- 2\sin x \cos x + \cos^2 x \\ &= 2(\sin^2 x + \cos^2 x) \\ &= 2\end{aligned}$$

48.  $\sin x \cos x \sec x \cot x = \sin x \cos x \left(\frac{1}{\cos x}\right) \left(\frac{\cos x}{\sin x}\right)$   
 $= \cos x$

49.  $\cos x \tan x + \sin x \cot x = \cos x \left(\frac{\sin x}{\cos x}\right) + \sin x \left(\frac{\cos x}{\sin x}\right)$   
 $= \sin x + \cos x$

50.  $(1 + \cos \theta)(\csc \theta - \cot \theta) = (1 + \cos \theta) \left(\frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta}\right)$   
 $= (1 + \cos \theta) \left(\frac{1 - \cos \theta}{\sin \theta}\right)$   
 $= \frac{1 - \cos^2 \theta}{\sin \theta}$   
 $= \frac{\sin^2 \theta}{\sin \theta}$   
 $= \sin \theta$

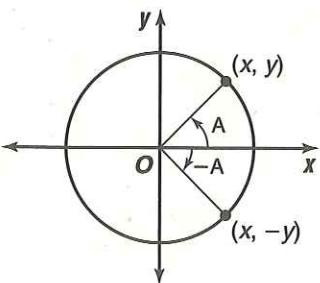
51.  $1 + \cot^2 \theta - \cos^2 \theta - \cos^2 \theta \cot^2 \theta$   
 $= 1 + \cot^2 \theta - \cos^2 \theta(1 + \cot^2 \theta)$   
 $= \csc^2 \theta - \cos^2 \theta(\csc^2 \theta)$   
 $= \csc^2 \theta(1 - \cos^2 \theta)$   
 $= \csc^2 \theta(\sin^2 \theta)$   
 $= \frac{1}{\sin^2 \theta} (\sin^2 \theta)$   
 $= 1$

52.  $\frac{\sin x}{1 + \cos x} + \frac{\sin x}{1 - \cos x}$   
 $= \frac{\sin x - \sin x \cos x}{1 - \cos^2 x} + \frac{\sin x + \sin x \cos x}{1 - \cos^2 x}$   
 $= \frac{2 \sin x}{1 - \cos^2 x}$   
 $= \frac{2 \sin x}{\sin^2 x}$   
 $= \frac{2}{\sin x}$   
 $= 2 \csc x$

53.  $\cos^4 \alpha + 2\cos^2 \alpha \sin^2 \alpha + \sin^4 \alpha = (\cos^2 \alpha + \sin^2 \alpha)^2$   
 $= 1^2 \text{ or } 1$

54.  $I = I_0 \cos^2 \theta$   
 $0 = I_0 \cos^2 \theta$   
 $0 = \cos^2 \theta$   
 $0 = \cos \theta$   
 $\cos^{-1} 0 = \theta$   
 $90^\circ = \theta$

55. Let  $(x, y)$  be the point where the terminal side of  $A$  intersects the unit circle when  $A$  is in standard position. When  $A$  is reflected about the  $x$ -axis to obtain  $-A$ , the  $y$ -coordinate is multiplied by  $-1$ , but the  $x$ -coordinate is unchanged. So,  $\sin(-A) = -y = -\sin A$  and  $\cos(-A) = x = \cos A$ .



56a.  $e = \frac{W \sec \theta}{As}$

$eAs = W \sec \theta$

$\frac{eAs}{\sec \theta} = W$

$W = eAs \cos \theta$

56b.  $W = eAs \cos \theta$

$W = 0.80(0.75)(1000) \cos 40^\circ$

$W \approx 459.6266659$

459.63 W

57.  $F_N - mg \cos \theta = 0$

$F_N = mg \cos \theta$

$mg \sin \theta - \mu_k F_N = 0$

$mg \sin \theta - \mu_k (mg \cos \theta) = 0$

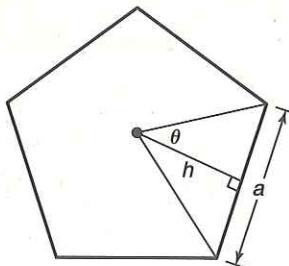
$\mu_k (mg \cos \theta) = mg \sin \theta$

$\mu_k = \frac{mg \sin \theta}{mg \cos \theta}$

$\mu_k = \frac{\sin \theta}{\cos \theta}$

$\mu_k = \tan \theta$

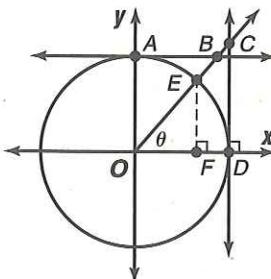
58.



$\theta = \frac{360^\circ}{2n} = \frac{180^\circ}{n}$ ,  $\tan \theta = \frac{\frac{a}{2}}{h}$ , so  $h = \frac{a}{2 \tan \theta} = \frac{a}{2} \cot \theta$ .

The area of the isosceles triangle is  $\frac{1}{2}(a)\left(\frac{a}{2} \cot \frac{180^\circ}{n}\right) = \frac{a^2}{4} \cot \left(\frac{180^\circ}{n}\right)$ . There are  $n$  such triangles, so  $A = \frac{1}{4}na^2 \cot \left(\frac{180^\circ}{n}\right)$ .

59.



$\sin \theta = EF$  and  $\cos \theta = OF$  since the circle is a unit circle.  $\tan \theta = \frac{CD}{OD} = \frac{CD}{1} = CD$ .

$\sec \theta = \frac{CO}{OD} = \frac{CO}{1} = CO$ .  $\triangle EOF \sim \triangle OBA$ , so

$\frac{OF}{EF} = \frac{BA}{OA} = \frac{BA}{1} = BA$ . Then  $\cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{OF}{EF} = BA$ .

Also by similar triangles,  $\frac{EO}{EF} = \frac{OB}{OA}$ , or  $\frac{1}{EF} = \frac{OB}{1}$ .

Then  $\csc \theta = \frac{1}{\sin \theta} = \frac{1}{EF} = \frac{OB}{1} = OB$ .

60.  $\cos^{-1} \left(-\frac{\sqrt{2}}{2}\right) = 135^\circ$