Trigonometry
Review 7.1-7.4

Name Key

1. If $\sin\theta = \frac{2}{5}$, $90° < \theta < 180°$, find $\sec\theta$.

$\left(\frac{2}{5}\right)^2 + \cos^2\theta = 1$

$\frac{4}{25} + \cos^2\theta = \frac{25}{25}$

$\cos^2\theta = \frac{21}{25}$

$\cos\theta = -\frac{\sqrt{21}}{5}$

$\sec\theta = -\frac{5\sqrt{21}}{21}$

2. If $\cot\theta = \frac{5}{8}$, find $\tan\theta$.

$\frac{8}{5}$

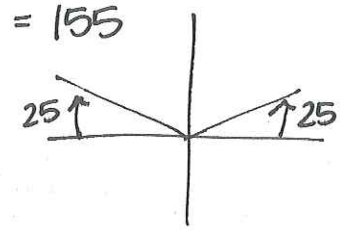
Express each value as a trigonometric function of an angle in Quadrant I.

3. $\tan\frac{13\pi}{3} - \frac{6\pi}{3} = \frac{7\pi}{3} - \frac{6\pi}{3} = \frac{\pi}{3}$

$\boxed{\tan\frac{\pi}{3}}$

4. $\sin 875° - 360 - 360 = 155$

$\boxed{\sin 25}$



Simplify each expression.

5. $\frac{\sec^2 x - 1}{\sec^2 x}$

$\frac{\tan^2 x}{\sec^2 x}$

$\frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}}$

$\frac{\sin^2 x}{\cos^2 x} \cdot \frac{\cos^2 x}{1}$

$\boxed{\sin^2 x}$

6. $\frac{\tan\alpha + \cot\alpha}{\tan\alpha}$

$\frac{\tan\alpha}{\tan\alpha} + \frac{\cot\alpha}{\tan\alpha}$

$1 + \frac{\frac{\cos\alpha}{\sin\alpha}}{\frac{\sin\alpha}{\cos\alpha}}$

$1 + \frac{\cos\alpha}{\sin\alpha} \cdot \frac{\cos\alpha}{\sin\alpha}$

$1 + \frac{\cos^2\alpha}{\sin^2\alpha}$

$1 + \cot^2\alpha$

$\boxed{\csc^2\alpha}$

Find a numerical value of one trig function of each x.

7. $\frac{1+\tan^2 x}{\sec x} = \sin^2 x + \frac{1}{\sec^2 x}$

$\frac{\sec^2 x}{\sec x} = \sin^2 x + \cos^2 x$

$\boxed{\sec x = 1}$

8. $(\sin\theta)(\sec\theta) = \frac{\sqrt{2}}{2}$

$\sin\theta \cdot \frac{1}{\cos\theta} = \frac{\sqrt{2}}{2}$

$\frac{\sin\theta}{\cos\theta} = \frac{\sqrt{2}}{2}$

$\boxed{\tan\theta = \frac{\sqrt{2}}{2}}$

Verify each identity.

9. $\tan 2\theta = \frac{2}{\cot\theta - \tan\theta}$

$\tan 2\theta = \frac{2}{\frac{\cos\theta}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} \cdot \frac{\sin\theta}{\sin\theta}}$

$= \frac{2}{\frac{\cos^2\theta - \sin^2\theta}{\cos\theta\sin\theta}}$

$= \frac{2}{\frac{\cos 2\theta}{\cos\theta\sin\theta}}$

$= 2 \cdot \frac{\cos\theta\sin\theta}{\cos 2\theta}$

$= \frac{\sin 2\theta}{\cos 2\theta}$

$\boxed{\tan 2\theta = \tan 2\theta}$

10. $1 + \frac{1}{2}\sin 2A = \frac{\sec A + \sin A}{\sec A}$

$1 + \frac{1}{2}(2\sin A\cos A) =$

$1 + \sin A\cos A = \frac{\frac{1}{\cos A} + \sin A \cdot \frac{\cos A}{\cos A}}{\frac{1}{\cos A}}$

$= \frac{\frac{1 + \sin A\cos A}{\cos A}}{\frac{1}{\cos A}}$

$= \frac{1 + \sin A\cos A}{\cos A} \cdot \frac{\cos A}{1}$

$\boxed{1 + \sin A\cos A = 1 + \sin A\cos A}$

11. cos (90 - θ) = sin θ

$\cos 90 \cos\theta + \sin 90 \sin\theta = \sin\theta$

$0 \cdot \cancel{\cos\theta} + 1 \cdot \sin\theta = \sin\theta$

$\sin\theta = \sin\theta$

12. $\tan(x + 45°) = \frac{1+\tan x}{1-\tan x}$

$\frac{\tan x + \tan 45}{1 - \tan x \tan 45} =$

$\frac{\tan x + 1}{1 - \tan x} = \frac{1 + \tan x}{1 - \tan x}$

Use sum or difference identities to find the exact value of each trigonometric function.

13. sin 255

$\sin(225 + 30)$

$\sin 225 \cos 30 + \sin 30 \cos 225$

$-\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot -\frac{\sqrt{2}}{2} = \boxed{\frac{-\sqrt{6} - \sqrt{2}}{4}}$

14. $\cos \frac{7\pi}{12}$

$\cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{4}\sin\frac{\pi}{3}$

$= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$

$= \boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$

15. Find exact value of tan (x – y) if sin x = $-\frac{5}{13}$ and cos y = $\frac{4}{5}$ and x and y are ~~angles in~~ quad IV.

$\frac{\tan x - \tan y}{1 + \tan x \tan y}$

$\frac{1}{3} \cdot \frac{48}{33}$

$\frac{48}{99}$

$1 + \cot^2 x = \left(-\frac{13}{5}\right)^2$

$1 + \cot^2 x = \frac{169}{25}$

$\cot x = -\frac{12}{5}$

$\tan x = -\frac{5}{12}$

$1 + \tan^2 y = \left(\frac{5}{4}\right)^2$

$1 + \tan^2 y = \frac{25}{16}$

$\tan y = -\frac{3}{4}$

$\frac{-\frac{5}{12} + \frac{3}{4}}{1 + \left(-\frac{3}{4}\right)\left(-\frac{5}{12}\right)} = \frac{\frac{4}{12}}{1 - \frac{15}{48}} = \frac{\frac{1}{3}}{\frac{33}{48}} = \frac{16}{33}$

16. If sec θ = -4 and 180° < θ < 270°, ~~then find~~ the exact value of cos 2θ, sin 2θ, and tan 2θ.

$\cos\theta = -\frac{1}{4}$

$\sin^2\theta + \left(-\frac{1}{4}\right)^2 = 1$

$\sin^2\theta + \frac{1}{16} = \frac{16}{16}$

$\sin\theta = -\sqrt{15}/4$

$\sin 2\theta = 2\left(-\frac{\sqrt{15}}{4}\right)\left(-\frac{1}{4}\right)$

$= \frac{2\sqrt{15}}{16}$

$= \frac{\sqrt{15}}{8}$

$\cos 2\theta = \left(-\frac{1}{4}\right)^2 - \left(-\frac{\sqrt{15}}{4}\right)^2$

$= \frac{1}{16} - \frac{15}{16}$

$= -\frac{14}{16} = -\frac{7}{8}$

$\tan 2\theta = \frac{\sqrt{15}/8}{-7/8}$

$= -\frac{\sqrt{15}}{7}$

Use a half-angle identity to find the exact value of each trigonometric function.

17. tan 112.5

$\tan \frac{225}{2}$

$-\sqrt{\frac{1 - -\frac{\sqrt{2}}{2}}{1 + -\frac{\sqrt{2}}{2}}}$

$-\sqrt{\frac{1 + \frac{\sqrt{2}}{2}}{1 - \frac{\sqrt{2}}{2}}} \cdot \sqrt{\frac{2}{2}}$

$-\sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}} \cdot \sqrt{\frac{2+\sqrt{2}}{2+\sqrt{2}}}$

$\rightarrow -\sqrt{\frac{(2+\sqrt{2})^2}{4-2}}$

$-\left(\frac{2+\sqrt{2}}{\sqrt{2}}\right)\frac{\sqrt{2}}{\sqrt{2}}$

$-\left(\frac{2\sqrt{2}+2}{2}\right)$

$-(\sqrt{2}+1)$

$\boxed{-\sqrt{2} - 1}$

18. $\sin \frac{5\pi}{8}$

$\sin \frac{5\pi/4}{2} = \sqrt{\frac{1 + \frac{\sqrt{2}}{2}}{2}}$

$= \sqrt{\frac{1 + \frac{\sqrt{2}}{2}}{2}} \cdot \sqrt{\frac{2}{2}}$

$= \sqrt{\frac{2+\sqrt{2}}{4}}$

$= \boxed{\frac{\sqrt{2+\sqrt{2}}}{2}}$