

Rationalizing the Denominator

Recall:

$$\sqrt{a}\sqrt{b} = \sqrt{ab} \text{ for } a \geq 0 \text{ and } b \geq 0.$$

$$(a - b)(a + b) = a^2 - b^2 \text{ for any } a \text{ and } b. (a - b) \text{ and } (a + b) \text{ are conjugates.}$$

I. Model Problems

In these examples we will practice rationalizing the denominator.

Example 1: $\frac{4}{\sqrt{6}}$

Multiply by a convenient form of 1.

$$\frac{4}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} \\ \frac{4\sqrt{6}}{6} \\ \frac{2\sqrt{6}}{3}$$

Simplify.

Reduce.

Answer: $\frac{2\sqrt{6}}{3}$

Example 2: $\frac{6}{7+\sqrt{5}}$

Find the conjugate of $7 + \sqrt{5}$.

Multiply by a convenient form of 1 with the conjugate.

$$\frac{6}{7+\sqrt{5}} \cdot \frac{7-\sqrt{5}}{7-\sqrt{5}} \\ \frac{42-6\sqrt{5}}{49-5} \\ \frac{42-6\sqrt{5}}{44} \\ \frac{21-3\sqrt{5}}{22}$$

Simplify.

Reduce.

Answer: $\frac{21-3\sqrt{5}}{22}$

II. Practice Problems

Rationalize the denominators.

1. $\frac{2}{\sqrt{3}}$

2. $-\frac{8}{\sqrt{6}}$

3. $\frac{5}{\sqrt{10}}$

4. $\frac{12}{\sqrt{15}}$

5. $\frac{5}{3\sqrt{7}}$

6. $\frac{10}{3\sqrt{2}}$

7. $-\frac{21}{10\sqrt{3}}$

8. $\frac{3}{4+\sqrt{5}}$

9. $\frac{5}{6+\sqrt{3}}$

10. $\frac{5}{\sqrt{7}+4}$

11. $\frac{3\sqrt{5}}{9-\sqrt{5}}$

12. $\frac{2\sqrt{7}}{4+\sqrt{6}}$

13. $\frac{3}{3-\sqrt{3}}$

14. $\frac{5}{\sqrt{3}+\sqrt{2}}$

15. $\frac{12}{4-\sqrt{2}}$

16. $\frac{4\sqrt{5}}{-4+\sqrt{5}}$

17. $\frac{1}{3\sqrt{2}}$

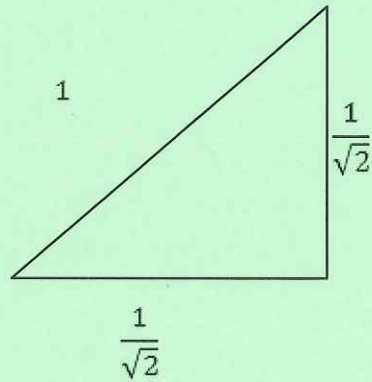
18. $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}$

19. $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

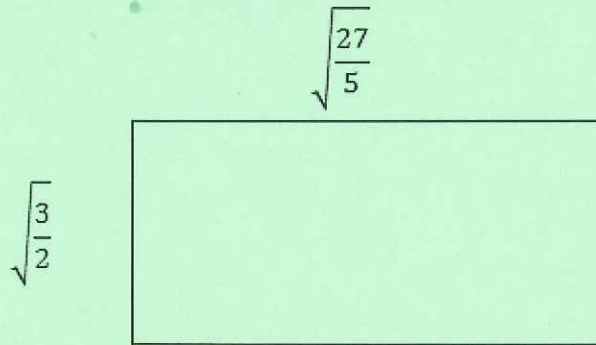
20. $\frac{\sqrt{15}+\sqrt{6}}{\sqrt{15}-\sqrt{6}}$

III. Challenge Problems

1. Find the perimeter of the triangle below.



2. Find the area of the rectangle below.



3. Find the mistake in the students work.

$$\frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4} = \frac{\sqrt{1}}{2} = \frac{1}{2}$$

4. Find the mistake in the students work.

$$\frac{8 + \sqrt{2}}{7 - \sqrt{2}} \cdot \frac{7 + \sqrt{2}}{7 + \sqrt{2}} = \frac{48 + 2}{49 - 2} = \frac{50}{47}$$

IV. Answer Key

1. $\frac{2\sqrt{3}}{3}$
2. $-\frac{4\sqrt{6}}{3}$
3. $\frac{\sqrt{10}}{2}$
4. $\frac{4\sqrt{15}}{5}$
5. $\frac{5\sqrt{7}}{21}$
6. $\frac{10\sqrt{2}}{6}$
7. $\frac{7\sqrt{3}}{10}$
8. $\frac{12-3\sqrt{5}}{11}$
9. $\frac{30-5\sqrt{3}}{33}$
10. $\frac{5\sqrt{7}-20}{-9}$
11. $\frac{15+27\sqrt{5}}{76}$
12. $\frac{4\sqrt{7}+\sqrt{42}}{2}$
13. $\frac{3+\sqrt{3}}{2}$
14. $5\sqrt{3} - 5\sqrt{2}$
15. $\frac{24+6\sqrt{2}}{7}$
16. $\frac{-16\sqrt{5}-20}{11}$
17. $\frac{\sqrt{2}}{6}$
18. $-5 - 2\sqrt{6}$
19. $4 - \sqrt{15}$
20. $\frac{7+2\sqrt{10}}{3}$

Challenge Problems

1. $1 + \sqrt{2}$
2. $\frac{9\sqrt{10}}{10}$
3. Numbers inside a radical can not reduce with numbers outside the radical. The answer is $\frac{\sqrt{2}}{4}$.
4. Must FOIL the numerators. $\frac{8+\sqrt{2}}{7-\sqrt{2}} \cdot \frac{7+\sqrt{2}}{7+\sqrt{2}} = \frac{56+8\sqrt{2}+7\sqrt{2}+2}{49-2} = \frac{58+15\sqrt{2}}{47}$