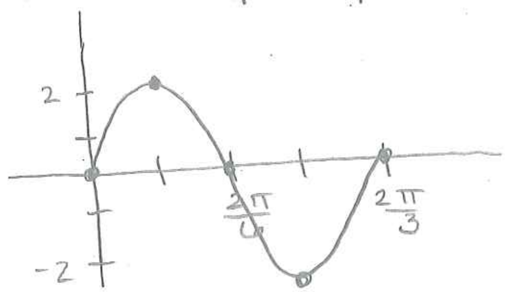
**Trig/Pre-Calc** **Name** Key

## More Practice for sections 6.1 – 6.6

For numbers 1 – 6, determine the amplitude, period, frequency, phase shift, and vertical translation for each. Describe the transformations required to obtain the trig function starting from the parent function.

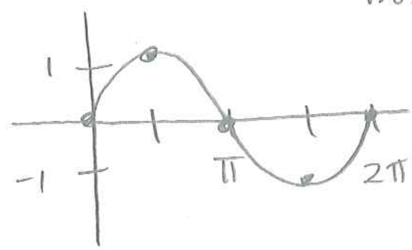
1. $y =$ $2\sin 3x$   Amp = 2   per = $\frac{2\pi}{3}$   P.S. + V.S. = 0




2. $y = -\sin(x - \pi)$   Amp = 1   Per = $2\pi$

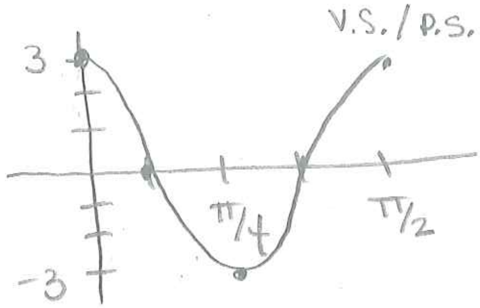
P.S. = $\frac{\pi}{1} = \pi$   V.S. = 0

| X | Y |
|---|---|
| $\pi/2$ | 1 |
| $\pi$ | 0 |
| $3\pi/2$ | −1 |
| $2\pi$ | 0 |
| 0 | 0 |



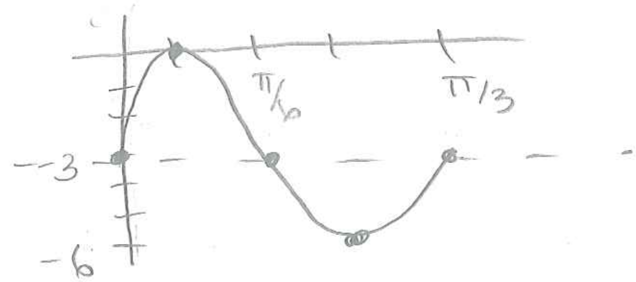

3. $y = 3\cos 4x$   Amp = 3   Per = $\frac{2\pi}{4} = \frac{\pi}{2}$

V.S. / P.S. = 0




4. $y = 3\sin 6x - 3$

Amp = 3   per = $\frac{2\pi}{6} = \frac{\pi}{3}$   V.S. = −3   P.S. = 0

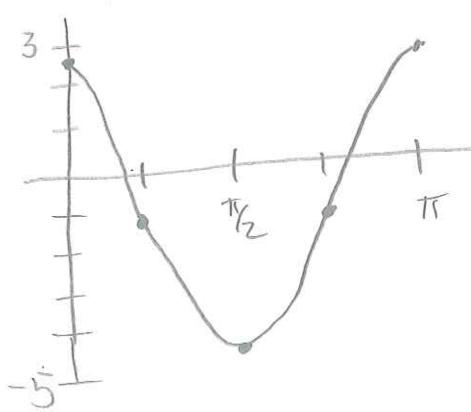



5. $y = -3.5\sin(2x - \frac{\pi}{2}) - 1$

Amp = 3.5   V.S. = −1   per = $\frac{2\pi}{2} = \pi$

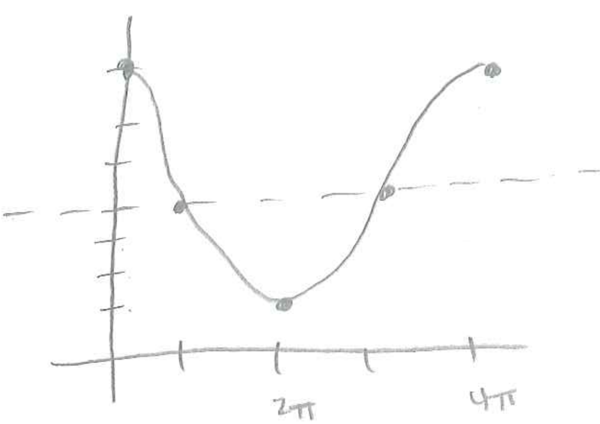
P.S. = $\frac{\pi/2}{2} = \pi/2 \cdot \frac{1}{2} = \pi/4$

| X | Y |
|---|---|
| 0 | 2.5 |
| $\pi/4$ | −1 |
| $\pi/2$ | −4.5 |
| $3\pi/4$ | −1 |
| $\pi$ | 2.5 |




6. $y = 3\cos\frac{1}{2}x + 4$   Amp = 3   VS. = 4

Per = $\frac{2\pi}{1/2} = 4\pi$   P.S. = 0

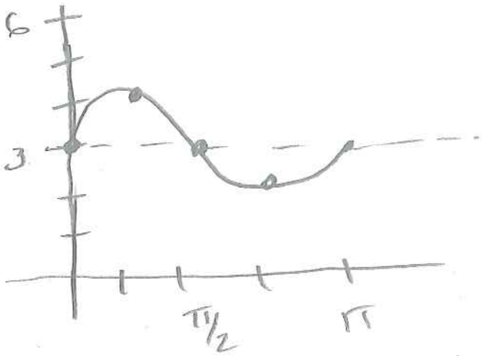


For numbers 7 – 10, sketch the graph of each function for one period.

7. $y = \sin 2x + 3$

Amp = 1  V.S. = 3  P.S. = 0

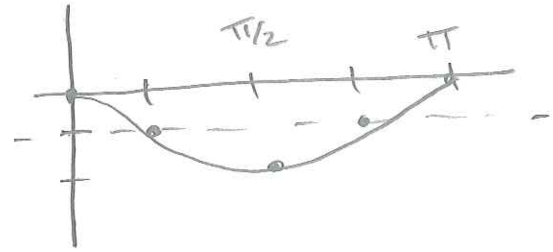
per = $\frac{2\pi}{2} = \pi$




8. $y = \cos 2x - 1$

Amp = 1  V.S. = −1

PS = 0  per = $\frac{2\pi}{2} = \pi$

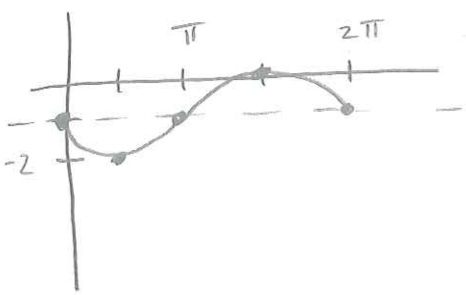



9. $y = \sin(x - \pi) - 1$

Amp = 1  V.S. = −1

per = 2π  P.S. = π

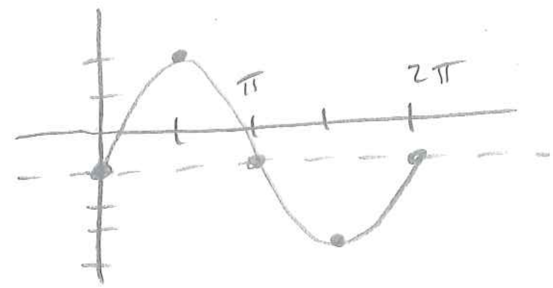
| x | y |
|---|---|
| 0 | −1 |
| π/2 | −2 |
| π | −1 |
| 3π/2 | 0 |
| 2π | −1 |




10. $y = 3\sin x - 1$

Amp = 3  V.S. = −1

P.S. = 0  per = 2π




11. Find an equation for a sine function that has an amplitude of 4, a period of 180°, and a y-intercept of −3.

↳ π

$y = 4 \sin 2\theta - 3$

$\frac{2\pi}{k} = \pi$

$\pi k = 2\pi$

$k = 2$

12. Find an equation for a cosine function that has an amplitude of $\frac{3}{5}$, a period of 270°, and a vertical shift of 5.

↓

$\frac{3\pi}{2}$

$y = \frac{3}{5} \cos \frac{4}{3}\theta + 5$

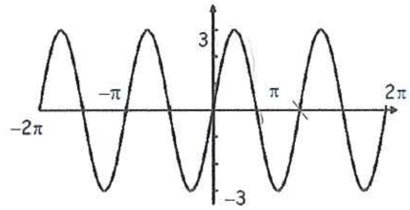
per: $\frac{2\pi}{k} = \frac{3\pi}{2}$

$\frac{3\pi k}{3\pi} = \frac{4\pi}{3\pi}$

$k = 4/3$

For numbers 13-16, identify the amplitude and period of each function graphed below. Then write an equation of each graph.

13.



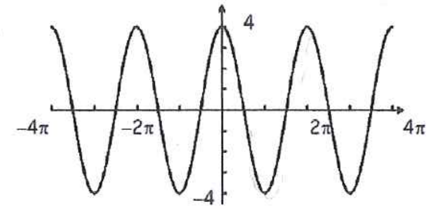

Amp = 3 Per = $\pi$ $\frac{2\pi}{k} = \pi$

$\pi k = 2\pi$

$k = 2$

$y = 3\sin 2\theta$

14.



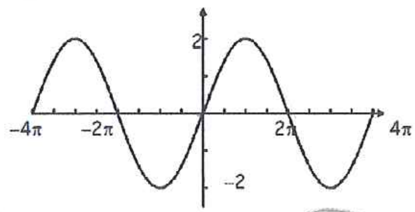

amp = 4

Per = $2\pi$

k = 1

$y = 4\cos\theta$

15.




Amp = 2

Per = $4\pi$

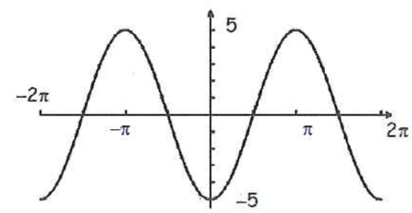
$\frac{2\pi}{k} = 4\pi$

$4\pi k = 2\pi$

$k = \frac{1}{2}$

$y = 2\sin \frac{1}{2}\theta$

16.




Amp = 5

Per = $2\pi$

k = 1

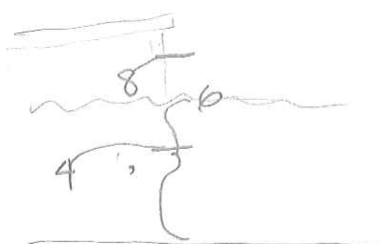
$y = -5\cos\theta$

V.S. = 6

17. The average depth of water at the end of a dock is 6 feet. This varies 2 feet in both directions with the tide. Suppose there is a high tide at 4 AM. If the tide goes from low to high every 6 hours, write a cosine function d(t) describing the depth of the water as a function of time with t = 4 corresponding to 4 AM.

V.S. = 6
Amp = 2

+




Per = 6 P.S. = 4

$\frac{2\pi}{k} = 6$ $\frac{c}{\pi/3} = 4$

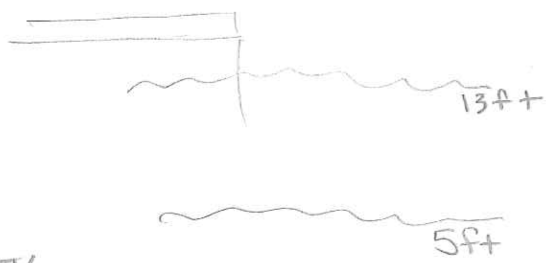
$6k = 2\pi$

$k = \pi/3$ $c = \frac{4\pi}{3}$

$y = \cancel{4}2\cos\left(\frac{\pi}{3} - \frac{4\pi}{3}\right) + 6$

18. The depth of water at a boat dock varies with the tides. The depth is 5 feet at low tide and 13 feet at high tide. On a certain day, low tide occurs at 4 am and high tide occurs at 10am. If $y$ represents the depth of the water $x$ hours after midnight, use a sine function to model the water's depth.

$\text{Amp} = \frac{13-5}{2} = 4 \qquad \text{V.S.} = \frac{13+5}{2} = 9$




$\text{per} = 6 \qquad \frac{2\pi}{k} = 6 \qquad 6k = 2\pi$

$k = \pi/3$

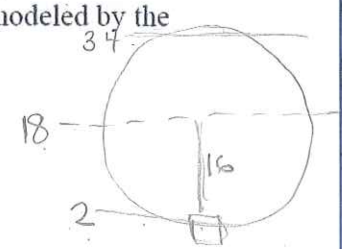
$\text{P.S.} = 4$

$\frac{c}{\pi/3} = 4$

$c = 4\pi/3$

$y = -4\sin\left(\frac{\pi}{3}\theta - \frac{4\pi}{3}\right) + 9$

19. The height, $h$, in meters, above the ground of a car as a Ferris Wheel rotates can be modeled by the function $h(t) = 16\cos\left(\frac{\pi t}{120}\right) + 18$, where $t$ is the time, in seconds.




I. What is the radius of the Ferris Wheel?

(A) 18 m (B) 8 m (C) 16 m (D) 9 m (E) 2 m

II. What is the maximum height of the car?

(A) 18 m (B) 26 m (C) 16 m (D) 17 m (E) 34 m

III. How long does it take for the wheel to make one revolution?

$k = \frac{\pi}{120}$

(A) 120 s (B) $\frac{\pi}{120}$ s (C) 240 s (D) $\frac{120}{\pi}$ s (E) 60 s

$\text{per} = \frac{2\pi}{\pi/120} = 2\pi \cdot \frac{120}{\pi} = 240$

IV. What is the minimum height of the car?

(A) 2 m (B) 4 m (C) 16 m (D) 18 m (E) 0 m

V. How fast does the wheel rotate?

(A) 120 rad/s (B) $\frac{\pi}{120}$ rad/s (C) 240 rad/s (D) $\frac{\pi}{240}$ rad/s (E) $\frac{120}{\pi}$ rad/s

20. The average monthly temperatures in degrees Fahrenheit for the city of Las Vegas are given below.

| Jan | Feb | March | April | May | June | July | Aug | Sept | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 57°f | 63°f | 70°f | 79°f | 88°f | 100°f | 106°f | 102°f | 95°f | 82°f | 66°f | 57°f |

a. Write a sinusoidal function that models the monthly temperatures, using t = 1 to represent January.

Amp $= \frac{106-57}{2} = 24.5$

V.S. $= \frac{106+57}{2} = 81.5$

Per = 12 $\quad \frac{2\pi}{k} = 12$

$12k = 2\pi$

$k = \pi/6$

$y = -24.5 \cos(\pi/6\, t - c) + 81.5$

$57 = -24.5 \cos(\pi/6 - c) + 81.5$

$-24.5 = -24.5 \cos(\pi/6 - c)$

$1 = \cos(\pi/6 - c)$

$\cos^{-1}(1) = \pi/6 - c$

$c = \pi/6 - \cos^{-1}(1) = \pi/6$

$y = -24.5 \cos(\pi/6\, t - \pi/6) + 81.5$

b. According to your model, what is the average monthly temperature in August? How does this compare to the actual average temperature?

$t = 8$

$y = -24.5 \cos\left(\frac{\pi}{6} \cdot 8 - \pi/6\right) + 81.5$

$y = 102.7$

compares to 102

21. A buoy, bobbing up and down in the water as waves move past it, moves from its highest point to its lowest point and back to its highest point every 10 seconds. The distance between its highest and lowest point is 3 feet.

period = 10

$\frac{2\pi}{k} = 10$

$10k = 2\pi$

$k = \pi/5$

Amp = 1.5

$y = 1.5 \cos \pi/5\, t$

22. If the equilibrium point is $y = 0$, then $y = -5\cos\frac{\pi}{6}x$ models a buoy bobbing up and down in the water.

a. Where is the buoy at $t = 0$? At $t = 1$?

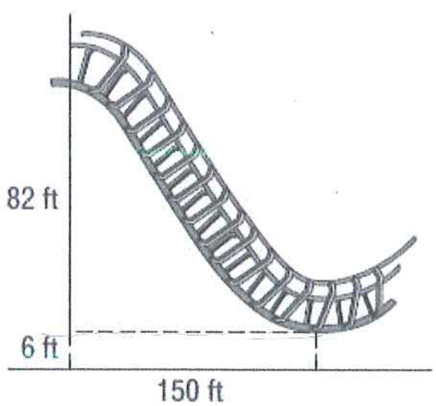
$t=0 \quad -5 \qquad t=1 \quad -4.33$

b. What is the maximum height of the buoy? The minimum?

$5 \qquad -5$

c. What is the period? $\frac{2\pi}{\pi/6} = 2\pi \cdot 6/\pi = 12$

20. **ROLLER COASTER** Part of a roller coaster track is a sinusoidal function. The high and low points are separated by 150 feet horizontally and 82 feet vertically as shown. The low point is 6 feet above the ground.




$\text{Amp} = \frac{82-6}{2} = 38$

$\text{V.S.} = \frac{82+6}{2} = 44$

a. Write a sinusoidal function that models the distance the roller coaster track is above the ground at a given horizontal distance $x$.

$\text{per} = 150$

$y = 38\cos\frac{\pi}{75}t + 44$

$\frac{2\pi}{k} = 150$

$k = \pi/75$

b. Point $A$ is 40 feet to the right of the $y$-axis. How far above the ground is the track at point $A$?

$t = 40$

approx 40 feet above ground