Strategy for factoring polynomials:

Step 1. ***GCF:*** If the polynomial has a greatest common factor other than 1, then factor out the greatest common factor.

Step 2. ***Binomials:*** If the polynomial has two terms (it is a binomial), then see if it is the ***difference of two squares***: $(a^2 - b^2)$.

Remember if it is the sum of two squares, it will NOT factor.

Step 3. ***Trinomials:*** If the polynomial is a trinomial, then check to see if it is a perfect square trinomial which will factor into the square of a binomial: $(a+b)^2$ *or* $(a-b)^2$.

- ❖ If it is not a perfect square trinomial, use factoring **by trial and error** or the AC method.

> ❖ **Strategy for factoring $ax^2 + bx + c$ by grouping (AC method):**
> a. Form the product ac
> b. Find a pair of numbers whose product is ac and whose sum is b.
> c. Rewrite the polynomial so that the middle term (bx) is written as the sum of two terms whose coefficients are the two numbers found in step 2.
> d. Factor by Grouping (as in step 4)

Step 4. ***Other polynomials***: If it has more than three terms, try to factor it by grouping.

a. Group two terms together which can be factored further
b. Use the distributive property in reverse to factor out common terms
c. Write the factors as multiplication of binomials.

Step 5. ***Final check***: See if any of the factors you have written can be factored further. If you have overlooked a common factor, you can catch it here.

Remember the following properties:	
Perfect Squares:	$(a+b)^2 = a^2 + 2ab + b^2$ and $(a-b)^2 = a^2 - 2ab + b^2$
Difference of two squares:	$a^2 - b^2 = (a-b)(a+b)$
Sum of two squares:	$a^2 + b^2$ is **NOT factorable**

Factoring, among other benefits, helps us simplify division of polynomials such as:

$$\frac{x^2 - 4}{x - 2}$$

Instead of trying to do the long division, let's see if we can factor the numerator so we can cancel some things out:

$$\frac{x^2 - 4}{x - 2} = \frac{(x-2)(x+2)}{(x-2)} = x + 2$$

Example:	Description of steps:
$2x^5 - 8x^3 =$ $2x^3(x^2 - 4) =$ $2x^3(x+2)(x-2)$	Step 1: Factor out greatest common factor $(2x^3)$ Step 2: Determine if the remaining binomial is the difference of two squares Step 2: It is the difference of two squares (skip steps 3-4) Step 5: Can it be factored further? No
$3x^4 - 18x^3 + 27x^2 =$ $3x^2(x^2 - 6x + 9) =$ $3x^2(x-3)^2$	Step 1: Factor out greatest common factor $(3x^2)$ Step 2: Determine if the remaining binomial is the difference of two squares: NOT binomial. Step 3: Determine if the remaining trinomial is a perfect square: It seems to be $(x-3)^2$ Step 5: Can it be factored further? No
$6a^2 - 11a + 4 =$ $6a^2 - 3a - 8a + 4 =$ $(6a^2 - 3a) + (-8a + 4) =$ $3a(2a-1) + (-4)(2a-1) =$ $(3a-4)(2a-1)$	Step 1: no GCF Step 2: Not a binomial Step 3: Not a perfect square; factor by AC method (or trial & error). a. Find the product of ac (24). b. Find two numbers whose product is ac (24) and whose sum is b (-11). The two numbers are -8 and -3. c. Rewrite the trinomial so the middle term is the sum of the two numbers found as coefficients. Step 4: Factor by grouping. Step 5: Cannot be factored further.
$xy + 8x + 3y + 24 =$ $(xy + 8x) + (3y + 24) =$ $x(y+8) + 3(y+8) =$ $(x+3)(y+8)$	Skip steps 1-3. Step 4: Factor by grouping a. group two terms together b. find GCF of each group c. Use distributive property to "pull out" the common term. d. Rewrite as product of two binomials Step 5: Cannot be factored further
$2ab^5 + 8ab^4 + 2ab^3 =$ $2ab^3(b^2 + 4b + 1)$	Step 1: Find GCF ($2ab^3$) Skip step 2 (not a binomial remaining) Step 3-4: Not a perfect square and can't be factored. Step 5: Cannot be factored further.
$x^2 + 5x + 6 =$ $(x+3)(x+2)$	Skip steps 1-2 Step 3: Not a perfect square, coefficient of first term is 1, so just reverse FOIL: a. First two terms are x and x b. Last two terms have to multiply to be 6 and sum to be 5. The two numbers are 2 and 3. c. Both signs need to be positive Step 4: Check the OI term to make sure it's correct. It is.

Factor the following polynomials using the strategy and examples above:

Polynomial:	Factored form:
$12a^2b^2 - 3ab$	
$4x^2 - 9$	
$x^2 - 16y^2$	
$x^2 - 4x + 2xy - 8y$	
$x^2 - 9x + 20$	
$9x^2 - 12x + 4$	
$8x^3 - x^2$	
$x^2 + 49$	
$16x^3 + 16x^2 + 3x$	
$x^2 - 9x + 18$	
$6x^2 + 13x + 6$	

$2x^2+3x-2$	
$5x^2-22x-15$	
$3x^3+9x^2-12x$	
$x^2+3x-28$	
$x^2-8x+16$	
$4x^2-7xy+3y^2$	
x^3-xy+x^2-y	
$8x^2-6x-2$	
$x^4-11x^3+24x^2$	
$6x^4y^5-2x^2y^3+14x^3y^4$	