

1. Find the distance and the midpoint between the points (-1, 2) and (3, 1).

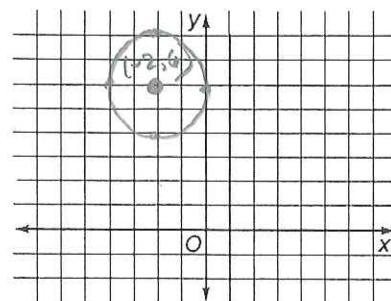
$$\frac{\sqrt{(3-(-1))^2 + (1-2)^2}}{\sqrt{16+1}} = \sqrt{17}$$

$$\left(\frac{-1+3}{2}, \frac{2+1}{2}\right) = (1, 1.5)$$

2. Write $x^2 + 4x + y^2 - 12y + 36 = 0$ in standard form. Then graph the equation, labeling the center.

$$x^2 + 4x + 4 + y^2 - 12y + 36 = -36 + 4 + 36$$

$$(x+2)^2 + (y-6)^2 = 4$$

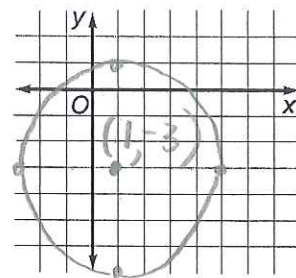


3. Write $6x^2 - 12x + 6y^2 + 36y = 36$ in standard form. Then graph the equation, labeling the center.

$$x^2 - 2x + y^2 + 6y = 6$$

$$x^2 - 2x + 1 + y^2 + 6y + 9 = 6 + 1 + 9$$

$$(x-1)^2 + (y+3)^2 = 16$$



4. Write $4x^2 + 16y^2 + 32x - 32y + 16 = 0$ in standard form. Then find the coordinates of the center, foci, and vertices of the ellipse. Graph the equation.

$$4x^2 + 32x + 16y^2 - 32y = -16$$

$$4(x^2 + 8x + 16) + 16(y^2 - 2y + 1) = -16 + 64 + 16$$

$$\frac{4(x+4)^2}{64} + \frac{16(y-1)^2}{64} = \frac{64}{64}$$

$$\frac{(x+4)^2}{16} + \frac{(y-1)^2}{4} = 1$$

center: (-4, 1)

vertices: (-4±4, 1) (-4, 1±2)
(0, 1) (-8, 1) (-4, 3) (-4, -1)

foci: (-4±2√3, 1)

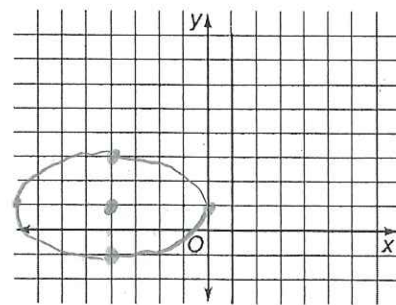
$$c^2 = a^2 - b^2$$

$$c^2 = 16 - 4$$

$$c^2 = 12$$

$$c = \sqrt{12}$$

$$c = 2\sqrt{3}$$



5. Write $4x^2 + 9y^2 + 9 = 24x - 18y$ in standard form. Then find the coordinates of the center, foci, and vertices of the ellipse. Graph the equation.

$$4x^2 - 24x + 9y^2 + 18y = -9$$

$$4(x^2 - 6x + 9) + 9(y^2 + 2y + 1) = -9 + 36 + 9$$

$$\frac{4(x-3)^2}{36} + \frac{9(y+1)^2}{36} = \frac{36}{36}$$

$$\frac{(x-3)^2}{9} + \frac{(y+1)^2}{4} = 1$$

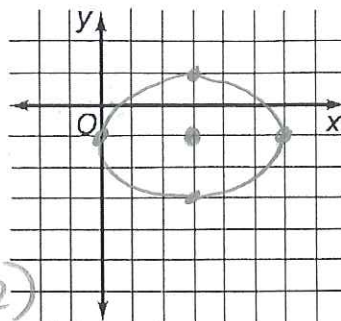
$$c^2 = 9 - 4$$

$$c = \sqrt{5}$$

center: $(3, -1)$

vertices: $(3 \pm 3, -1)$ $(3, -1 \pm 2)$
 $(6, -1)$ $(0, -1)$ $(3, 1)$ $(3, -3)$

foci: $(3 \pm \sqrt{5}, -1)$



6. Write $4x^2 - 25y^2 - 24x + 50y - 89 = 0$ in standard form. Find the coordinates of the center, foci, vertices and the equation of the asymptotes. Then graph the equation.

$$4x^2 - 24x - 25y^2 + 50y = 89$$

$$4(x^2 - 6x + 9) - 25(y^2 - 2y + 1) = 89 + 36 - 25$$

$$\frac{4(x-3)^2}{100} - \frac{25(y-1)^2}{100} = \frac{100}{100}$$

$$\frac{(x-3)^2}{25} - \frac{(y-1)^2}{4} = 1$$

center: $(3, 1)$

vertices: $(8, 1)$
 $(-2, 1)$

asympt: $(y-1) = \pm \frac{2}{5}(x-3)$

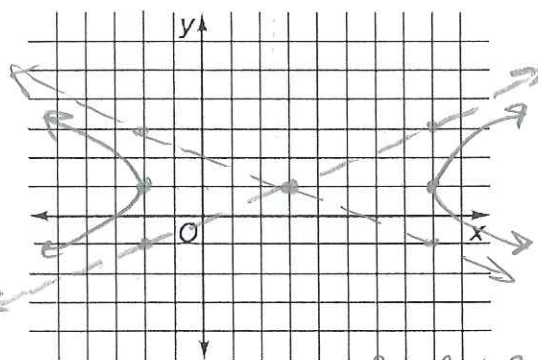
foci: $(3 \pm \sqrt{29}, 1)$

$$c^2 = a^2 + b^2$$

$$c^2 = 25 + 4$$

$$c^2 = 29$$

$$c = \sqrt{29}$$



7. Write $-2x^2 + 3y^2 - 24x - 6y - 93 = 0$ in standard form. Find the equations of the asymptotes of the graph. Then, graph the equation, labeling the center, foci, and vertices.

$$-2x^2 - 24x + 3y^2 - 6y = 93$$

$$-2(x^2 + 12x + 36) + 3(y^2 - 2y + 1) = 93 + 72 - 3$$

$$-2(x+6)^2 + 3(y-1)^2 = \frac{24}{24}$$

$$-\frac{(x+6)^2}{12} + \frac{(y-1)^2}{8} = 1$$

$$\frac{(y-1)^2}{8} - \frac{(x+6)^2}{12} = 1$$

$$a = 2\sqrt{2}$$

$$b = 2\sqrt{3}$$

center: $(-6, 1)$

vertices: $(-6, 1 \pm 2\sqrt{2})$

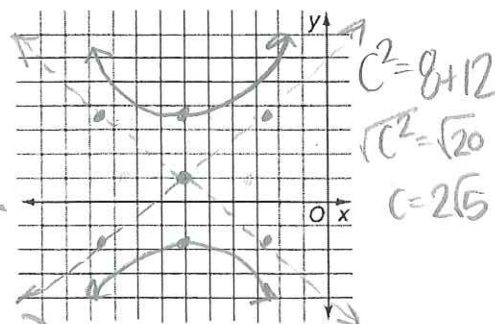
asympt: $(y-1) = \pm \frac{\sqrt{8}}{\sqrt{12}}(x+6)$

$$(y-1) = \pm \frac{\sqrt{6}}{3}(x+6)$$

foci: $(-6, 1 \pm 2\sqrt{5})$

$$\frac{\sqrt{8}}{\sqrt{12}} = \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{\sqrt{6}}{3}$$



8. Write $y^2 - 4x + 10y + 29 = 0$ in standard form. Find the coordinates of the focus, vertex, and the equation of the directrix.

$$y^2 + 10y = 4x - 29$$

$$y^2 + 10y + 25 = 4x - 29 + 25$$

$$(y+5)^2 = 4x - 4$$

$$(y+5)^2 = 4(x-1)$$

opens right

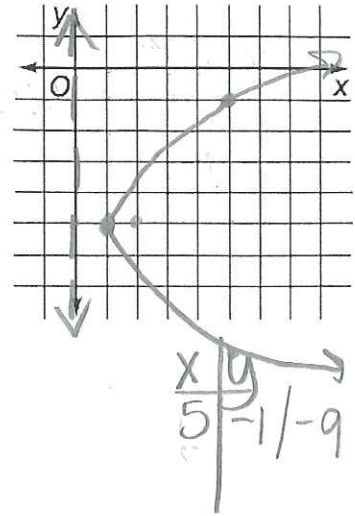
$$4p = 4$$

$$p = 1$$

$$\text{vertex: } (1, -5)$$

$$\text{focus: } (2, -5)$$

$$\text{directrix: } x = 0$$



$$(y+5)^2 = 10$$

$$y+5 = 4 \quad y+5 = -4$$

$$y = -1 \quad y = -9$$

9. Write $x = y^2 - 2y - 5$ in standard form. Find the equations of the directrix and axis of symmetry. Then, graph the equation, labeling the focus, vertex, and directrix.

$$x + 5 = y^2 - 2y + 1$$

$$x + 6 = (y-1)^2$$

opens right

$$4p = 1$$

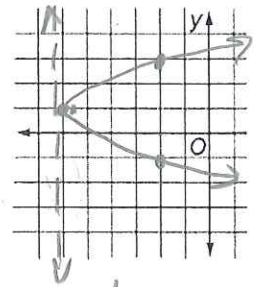
$$p = 1/4$$

$$\text{vertex: } (-6, 1)$$

$$\text{focus: } (-5.75, 1)$$

$$\text{directrix: } x = -6.25$$

$$\text{axis of symm: } y = 1$$



$$x/y = -2 \quad -1/3$$

$$4 = (y-1)^2$$

$$\pm 2 = y-1$$

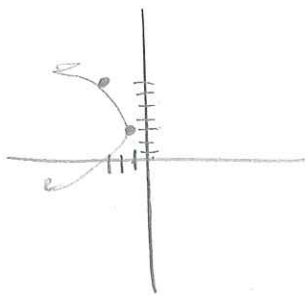
$$-2 = y-1$$

$$-1 = y$$

$$2 = y-1$$

$$3 = y$$

10. Find the equation of the parabola that passes through the point at $(-3, 7)$, has its vertex at $(-1, 3)$, and opens to the left.



$$(y-3)^2 = 4p(x+1)$$

$$(7-3)^2 = 4p(-3+1)$$

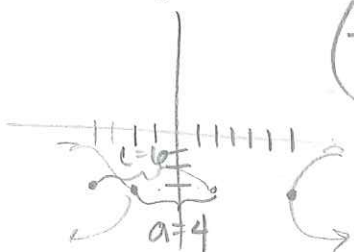
$$4^2 = 4p(-2)$$

$$16 = 4p(-2)$$

$$-8 = 4p$$

$$(y-3)^2 = -8(x+1)$$

11. Find the equation of the hyperbola that has vertices at $(-2, -3)$ and $(6, -3)$ and a focus is at $(-4, -3)$.



$$\left(\frac{-2+6}{2}, \frac{-3+(-3)}{2} \right) = (2, -3)$$

$$c^2 = a^2 + b^2$$

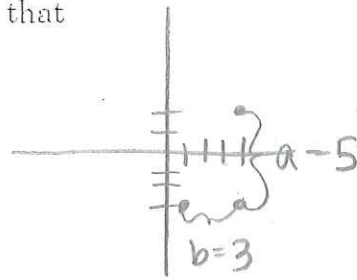
$$b^2 = 4^2 + b^2$$

$$20 = b^2$$

$$\frac{(x-2)^2}{16} - \frac{(y+3)^2}{20} = 1$$

12. Find the equation of the ellipse that has its major axis parallel to the y-axis and its center at $(4, -3)$, and that passes through points at $(1, -3)$ and $(4, 2)$.

$$\frac{(x-4)^2}{9} + \frac{(y+3)^2}{25} = 1$$



13. Find the equation of the equilateral hyperbola that has its foci at $(-2, -3 - 2\sqrt{3})$ and $(-2, -3 + 2\sqrt{3})$, and whose conjugate axis is 6 units long.

$$\frac{(y+3)^2}{3} - \frac{(x+2)^2}{9} = 1$$

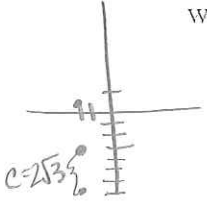
$$c^2 = a^2 + b^2$$

$$(2\sqrt{3})^2 = a^2 + 9$$

$$3 = a^2$$

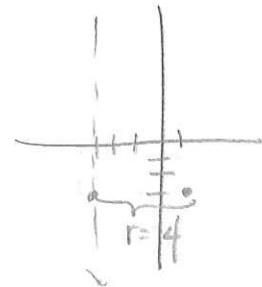
$$\left(\frac{-2-2}{2}, \frac{-3-2\sqrt{3} + -3+2\sqrt{3}}{2} \right)$$

$(-2, -3)$ - center



14. Write the standard form of the equation of the circle that is tangent to $x = -3$ and has its center at $(1, -3)$.

$$(x-1)^2 + (y+3)^2 = 16$$



15. Write the standard form of the equation of the circle that passes through $(-6, -4)$ and has its center at $(-8, 3)$.

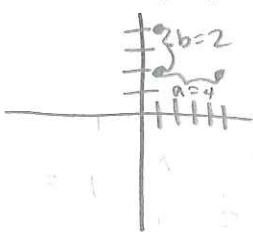
$$(x+8)^2 + (y-3)^2 = r^2$$

$$(-6+8)^2 + (-4-3)^2 = r^2$$

$$4 + 49 = r^2 \quad 53 = r^2$$

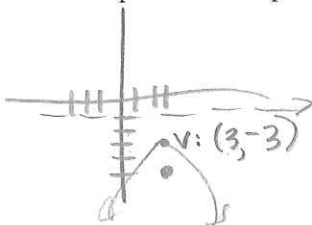
$$(x+8)^2 + (y-3)^2 = 53$$

16. Find the equation of the ellipse that has its center at $(1, 2)$, the major axis is parallel to the x-axis, and the ellipse passes through the points at $(1, 4)$ and $(5, 2)$.



$$\frac{(x-1)^2}{16} + \frac{(y-2)^2}{4} = 1$$

17. Write an equation of a parabola that has a focus at $(3, -5)$ and a directrix with equation $y = -1$.



$$(x-3)^2 = 4(-2)(y+3)$$

$$(x-3)^2 = -8(y+3)$$