Chapter 11 Test B   Pre-Calc   Name: _____Key_____

1. Evaluate $\left(16^{\frac{1}{2}} + 64^{\frac{1}{3}}\right)^{-\frac{1}{3}}$

   ③ $(4+4)^{-1/3}$

   $8^{-1/3}$ $\boxed{\frac{1}{2}}$

2. Simplify $\left(\frac{5x^{-3}y}{40\ 7}\right)^{\frac{2}{3}}$

   ③ $\left(\frac{1}{8x^3y^6}\right)^{2/3} = \boxed{\frac{1}{4x^2y^4}}$

3. Simplify $\sqrt{a^2b} \cdot \sqrt{a^3b^5}$

   ③ $\sqrt{a^5b^6} = |a|^2 b^3 \sqrt{a}$
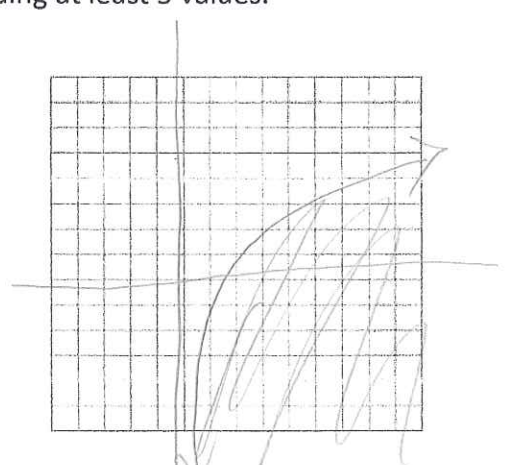
4. Express $\sqrt[4]{16xy^4}$ using rational exponents.

   ③ $16^{1/4} x^{1/4} y = 2x^{1/4} y$

5. Express $x^{\frac{1}{3}}(4x^3y^2)^{\frac{1}{2}}$ using only one type of radical.

   $x^{\frac{2}{6}} (4x^3y^2)^{\frac{3}{6}}$

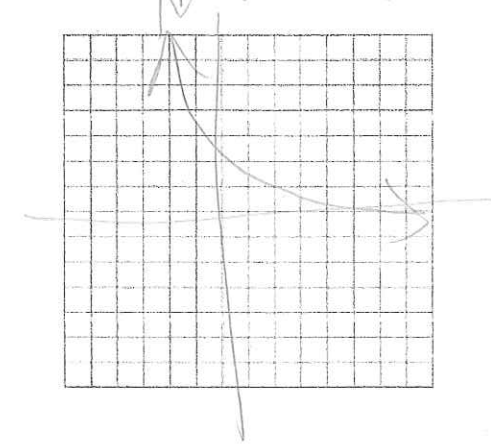   ④ $x^{2/6} 4^{3/6} x^{9/6} y^{6/6} = 2|x|y| \sqrt[6]{\cancel{16}4 x^5}$

   $4^{3/6} x^{11/6} y$

6. Sketch the graph of $y \leq \log_2 x$. Show your table of values including at least 5 values.

   ⑦

   -2 err
   -1 er
   0 ev
   1 0
   2 1
   4 2

7. Sketch the graph of $y = \left(\frac{1}{3}\right)^x$. Show your table of values including at least 5 values.

   ⑦

   -2 a
   -1 3
   0 1
   1 1/3
   2 1/9

8. Write $2^{-\frac{1}{3}} = 8$ in logarithmic form.

⟨2⟩ $\log_2 8 = -\frac{1}{3}$

9. Write $\log_4 256 = 4$ in exponential form.

⟨2⟩ $4^4 = 256$

10. Given that log 3 = 0.4771 and log 5 = 0.6990 find log 75. SHOW YOUR WORK, do not just plug into calc.

⟨3⟩ $\log(3 \cdot 5 \cdot 5) = \log 3 + 2\log 5$

$0.4771 + 2(0.6990)$

$\boxed{1.8751}$

Solve each equation or inequality. Round answers to the nearest ten-thousandth if necessary.

11. $2\log_5 3 + \frac{1}{5}\log_5 32 = \log_5(2x - 4)$

⟨5⟩ $3^2(32^{1/5}) = 2x - 4$

$18 = 2x - 4$

$2x = 22$

$\boxed{x = 11}$

12. $\log_9 x + \log_9(x - 2) = \log_9 3$

⟨5⟩ $x(x-2) = 3$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$\boxed{x = 3}$

13. $\log_3 28 - \log_3 x = \log_3 7$

⟨4⟩ $\frac{28}{x} = 7$

$\boxed{x = 4}$

14. $\log(10x) = \log(4x + 12)$

⟨4⟩ $10x = 4x + 12$

$6x = 12$

$\boxed{x = 2}$

15. $4^{2x-1} \geq 2^{x+3}$ ⟨5⟩

$(2x-1)\log 4 \geq (x+3)\log 2$

$2x\log 4 - \log 4 \geq x\log 2 + 3\log 2$

$2x\log 4 - x\log 2 \geq 3\log 2 + \log 4$

$x(2\log 4 - \log 2) \geq 3\log 2 + \log 4$

$\boxed{x \geq 1.6667}$

16. $5.4e^{0.28t} = 2e^{t+2}$ ⟨5⟩

$\ln 2.7 + 0.28t = t + 2$

$-1.0067 = 0.72t$

$\boxed{-1.3982 = t}$

-0.7267

17. $2e^{-0.5x} < 83.2$

$-0.5x < \ln 41.6$

(4) $x > -7.4562$

18. Evaluate $\log_{16} \frac{1}{8}$. $= x$

-0.75

(3)

$16^x = \frac{1}{8}$

$16^x = 8^{-1}$

$2^{4x} = 2^{-3}$

$4x = -3 \quad x = -\frac{3}{4}$

19. Compare an investment of \$2700 earning 8% interest for 5 years when it is compounded continuously versus one that is compounded quarterly.

$2700\, e^{0.08(5)}$

4027.93

$2700 \left(1 + \frac{0.08}{4}\right)^{20}$

4012.06

(5) cont is + 15.87 more

20. In 2000, the deer population in a certain area was 800. The number of deer increases exponentially at a rate of 7% per year. Predict the population in 2013.

(4)

$800 \left(1 + 0.07\right)^{13}$

1927.88   1928

21. What interest rate is required for an investment with continuously compounded interest to double in 8 years?

(4)

$\frac{\ln 2}{8} = 0.087$

8.7%

22. If you won \$250,000 in the lottery and you wanted that to accumulate to \$5 million for your retirement in 40 years what average annual investment rate would be necessary for you to invest at?

(5)

$5\,000\,000 = 250\,000\, e^{r\,40}$

$20 = e^{40r}$

$40r = \ln 20$

$r = 0.075$

7.5%

23. The data gives the number of bacteria, in millions, found in a certain culture.

Time (hours)	0	1	2	3	4
Bacteria	48	26	15	8	5

a. Find an exponential function that models the data.

① $y = 46.7906 (0.5654)^x$

b. Write the equation from part a in terms of base e.

② $y = 46.7906 (e^{\ln 0.5654 x})$

$y = 46.7906 \, e^{-0.5702 x}$

c. Use the model to estimate the half-time for the culture.

② $24 = 46.7906 \, e^{-0.5702 x}$

$\ln 0.5129 = -0.5702 x$

$x = 1.1709 \text{ hrs}$

Extra Credit

A. Solve $3^{x-3} \geq 2\sqrt[4]{4^{x-1}}$

+2

$x \leq 4.8434$

B. Taylor wishes to invest enough money now in order to have $100,000 in twenty years. How much less can she invest at 10% interest in an account in which the interest rate is compounded continuously rather than in one at the same rate that is compounded annually?

+2   14,864.36   13533.53

                    1330.83 less

C. A certain bacteria will triple in 6 hours. If the final count is 8 times the original count, how much time has passed.

+2

$3 N_0 = N_0 \, e^{KT}$

$\ln 3 = Kt$

$t = \frac{\ln 3}{K}$

$6 = \frac{\ln 3}{K}$

$K = 0.1831$

$8 N_0 = N_0 \, e^{KT}$

$8 = e^{0.1831 t}$

$t = 11.36$

| 11 hours 21 min |