/85   **Best wishes to:** _____Key_____

1. Evaluate $\left(81^{\frac{1}{2}} - 1^{-\frac{1}{3}}\right)^{\frac{1}{3}}$

③ $(9-1)^{\frac{1}{3}} = 8^{1/3}$

$= \boxed{2}$

2. Simplify $\left(\dfrac{3y^9}{48x^{-4}y}\right)^{-\frac{1}{4}}$

③ $\left(\dfrac{x^4 y^8}{16}\right)^{-1/4} = \dfrac{16^{\frac{1}{4}}}{xy^2} = \boxed{\dfrac{2}{xy^2}}$

3. Simplify $\sqrt{x^5 b} \cdot \sqrt{x^{-2}b^3}$

③ $\sqrt{x^3 b^4} = \boxed{|x|b^2 \sqrt{x}}$

4. Express $\sqrt[3]{27xy^2}$ using rational exponents.

③ $27^{1/3} x^{1/3} y^{2/3}$

$\boxed{3x^{1/3} y^{2/3}}$
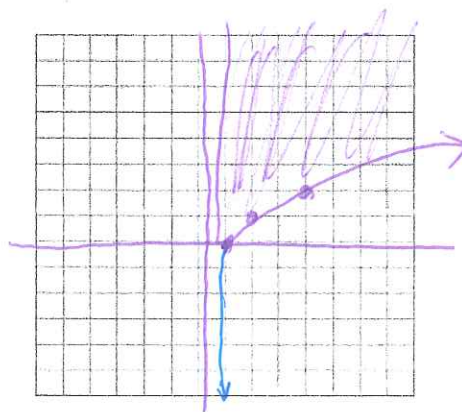
5. Express $a^{\frac{1}{5}}(7a^3 b)^{\frac{1}{2}}$ using only one type of radical.

④ $a^{2/10} (7a^3 b)^{5/10}$

$\sqrt[10]{a^2 7^5 a^{15} b^5} = \sqrt[10]{16807 a^{17} b^5} = |a| \sqrt[10]{16807 a^7 b^5}$

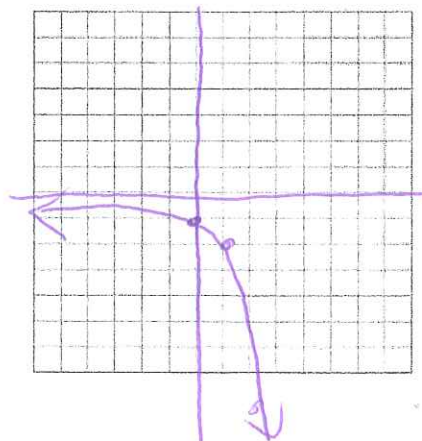6. Graph $y \geq \log_2 x$. Show your table of values including at least 5 values.

②

X	y
0	er
1	0
2	1
3	1.6
4	2

7. Graph $y = -3^x$. Show your table of values including at least 5 values.

②

X	Y
-1	-0.3
0	-1
1	-2
2	-9
3	-27

⓪

10

8. Write $36^{\frac{1}{2}} = 6$ in logarithmic form.

② $\log_{36} 6 = \frac{1}{2}$

9. Write $\log_7 343 = 3$ in exponential form.

② $7^3 = 343$

10. Given that log 7 = 0.8450 and log 3 = 0.4771 find log 0.063. SHOW YOUR WORK, do not just plug into calc.

③ $\log(7 \cdot 3^2 \cdot 10^{-3})$

$0.8450 + 2(0.4771) - 3 = \boxed{-1.2008}$

Solve each equation or inequality. Round answers to the nearest ten-thousandth if necessary.

11. $\log_5 4 + \frac{1}{4}\log_5 81 = \log_5(4x - 4)$

$\log_5 4(81)^{\frac{1}{4}} = \log_5(4x-4)$

⑤ $4(3) = 4x - 4$

$16 = 4x$

$\boxed{4 = x}$

12. $\log_9(x - 1) + \log_9(x - 2) = \log_9 6$

$\log_9(x-1)(x-2) = \log_9 6$

⑤ $x^2 - 3x + 2 = 6$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$\boxed{x = 4}$ ✗

13. $\log_3 x - \frac{1}{2}\log_3 25 = 2\log_3 10$

$\log_3 \frac{x}{5} = \log_3 100$

④ $\frac{x}{5} = 100$

$\boxed{x = 500}$

14. $\log(8x) = \log(2x + 18)$

④ $8x = 2x + 18$

$6x = 18$

$\boxed{x = 3}$

15. $2^{2x} \geq 5^{x-3}$

⑤ $2x \log 2 \geq (x-3) \log 5$

$2x \ln 2 \geq x \ln 5 - 3 \ln 5$

$2x \ln 2 - x \ln 5 \geq -3 \ln 5$

$x(2\ln 2 - \ln 5) \geq -3 \ln 5$

$\boxed{x \leq 21.6377}$

16. $4.1e^{0.5x} = 2e^{x-5}$

⑤ $2.05 e^{0.5x} = e^{x-5}$

$\ln 2.05 + 0.5x = x - 5$

$\ln 2.05 + 5 = -0.5x$

$\boxed{-11.4357 = x}$

17. $7e^{-1.8x} > 49.14$

$e^{-1.8x} > 7.02$

$-1.8x > \ln 7.02$

$\boxed{x < -1.0826}$

④

18. Evaluate $\log_6 \frac{2}{9}. = x$

③

$6^x = \frac{2}{9}$

$\dfrac{\log \frac{2}{9}}{\log 6}$

$\boxed{-0.8394}$

19. Compare an investment of $5400 earning 15% interest for 8 years when it is compounded continuously versus one that is compounded semi-annually.

⑤

$A = 5400 \left(1 + \frac{0.15}{2}\right)^{2(8)}$

$= 5400 (1.075)^{16}$

$= \boxed{17176.28}$
semi annual

$A = 5400\, e^{0.15(8)}$

$= \boxed{17,928.63}$

$\boxed{\text{Continuous} \rightarrow \text{more} \\ \$752.35}$

20. In 1994, the frog population in a certain area was 546. The number of frogs decreases exponentially at a rate of 2% per year. Predict the population in 2013.

④

$N = 546 (1 - 0.02)^{19}$

$= \boxed{372 \text{ frogs}}$

21. What interest rate is required for an investment with continuously compounded interest to double in 5 years?

④

$5 = \dfrac{\ln 2}{r}$

$r = \dfrac{\ln 2}{5} = 0.139$

$\boxed{13.9\%}$

22. If you won $250,000 in the lottery and you wanted that to accumulate to $6 million for your retirement in 40 years what average annual investment rate would be necessary for you to invest at?

⑤

$6,000,000 = 250,000\, e^{40r}$

$24 = e^{40r}$

$40r = \ln 24$

$r = 0.079$

$\boxed{7.9\%}$

23. The data gives the number of bacteria, in millions, found in a certain culture.

Time (hours)	0	1	2	3	4
Bacteria	48	26	15	8	5

(1)

a. Find an exponential function that models the data.

$$y = 46.7906 \, (0.5654)^x$$

(2)

b. Write the equation from part a in terms of base e.

$$y = 46.7906 \, (e^{\ln 0.5654 x})$$

$^{-0.5702}$

(2)

c. Use the model to estimate the **half-time** for the culture.

$$24 = 46.7906 \, e^{-0.5702x}$$

$$\ln 0.5129 = -0.5702x$$

$$\boxed{x = 1.1709 \text{ hours}}$$

Extra Credit

(+2)

A. Solve $3^{x-3} \geq 2\sqrt[4]{4^{x-1}}$

$$3^{x-3} \geq 2(4^{x-1})^{1/4}$$

$$(x-3)\ln 3 \geq \ln 2 + \tfrac{x-1}{4}\ln 4$$

$$x\ln 3 - 3\ln 3 \geq \ln 2 + \tfrac{x}{4}\ln 4 - \tfrac{1}{4}\ln 4$$

$$x\ln 3 - \tfrac{x}{4}\ln 4 \geq \ln 2 + 3\ln 3 - \tfrac{1}{4}\ln 4$$

$$x\left(\ln 3 - \tfrac{1}{4}\ln 4\right) \geq \ln 2 + 3\ln 3 - \tfrac{1}{4}\ln 4$$

$$\boxed{x \leq 4.8434}$$

B. Taylor wishes to invest enough money now in order to have $100,000 in twenty years. How much less can she invest at 10% interest in an account in which the interest rate is compounded continuously rather than in one at the same rate that is compounded annually?

(+2)

$$100\,000 = x\left(1 + \tfrac{.10}{1}\right)^{20}$$

$$14,864.36$$

$$100\,000 = xe^{.1(20)}$$

$$13533.53$$

$$\boxed{1330.83 \text{ less}}$$

C. A certain bacteria will triple in 6 hours. If the final count is 8 times the original count, how much time has passed.

(+2)

See other version

$$\tfrac{3x}{6} \quad \tfrac{8x}{y}$$

$$\boxed{\text{16 hours}}$$