7-6 Study Guide and Intervention (continued)

The Law of Sines

Use the Law of Sines to Solve Problems You can use the Law of Sines to solve
some problems that involve triangles.

Law of Sines	Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of the sides opposite the angles with measures A, B, and C, respectively. Then $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$.

Example Isosceles $\triangle ABC$ has a base of 24 centimeters and a
vertex angle of 68°. Find the perimeter of the triangle.

The vertex angle is 68°, so the sum of the measures of the base angles is
112 and $m\angle A = m\angle C = 56$.
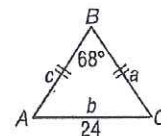
$\dfrac{\sin B}{b} = \dfrac{\sin A}{a}$ Law of Sines

$\dfrac{\sin 68°}{24} = \dfrac{\sin 56°}{a}$ $m\angle B = 68$, $b = 24$, $m\angle A = 56$

$a \sin 68° = 24 \sin 56°$ Cross multiply.

$a = \dfrac{24 \sin 56°}{\sin 68°}$ Divide each side by sin 68°.

≈ 21.5 Use a calculator.

The triangle is isosceles, so $c = 21.5$.

The perimeter is $24 + 21.5 + 21.5$ or about 67 centimeters.

Exercises

Draw a triangle to go with each exercise and mark it with the given information.
Then solve the problem. Round angle measures to the nearest degree and side
measures to the nearest tenth.

1. One side of a triangular garden is 42.0 feet. The angles on each end of this side measure
   66° and 82°. Find the length of fence needed to enclose the garden.

2. Two radar stations A and B are 32 miles apart. They locate an airplane X at the same
   time. The three points form $\angle XAB$, which measures 46°, and $\angle XBA$, which measures
   52°. How far is the airplane from each station?

3. A civil engineer wants to determine the distances from points A and B to an inaccessible
   point C in a river. $\angle BAC$ measures 67° and $\angle ABC$ measures 52°. If points A and B are
   82.0 feet apart, find the distance from C to each point.

4. A ranger tower at point A is 42 kilometers north of a ranger tower at point B. A fire at
   point C is observed from both towers. If $\angle BAC$ measures 43° and $\angle ABC$ measures 68°,
   which ranger tower is closer to the fire? How much closer?

7-6 Skills Practice

The Law of Sines

adds only!

Find each measure using the given measures from $\triangle ABC$. Round angle measures to the nearest tenth degree and side measures to the nearest tenth.

1. If $m\angle A = 35$, $m\angle B = 48$, and $b = 28$, find a.

2. If $m\angle B = 17$, $m\angle C = 46$, and $c = 18$, find b.

3. If $m\angle C = 86$, $m\angle A = 51$, and $a = 38$, find c.

4. If $a = 17$, $b = 8$, and $m\angle A = 73$, find $m\angle B$.

5. If $c = 38$, $b = 34$, and $m\angle B = 36$, find $m\angle C$.

6. If $a = 12$, $c = 20$, and $m\angle C = 83$, find $m\angle A$.

7. If $m\angle A = 22$, $a = 18$, and $m\angle B = 104$, find b.

Solve each $\triangle PQR$ described below. Round measures to the nearest tenth.

8. $p = 27$, $q = 40$, $m\angle P = 33$

9. $q = 12$, $r = 11$, $m\angle R = 16$

10. $p = 29$, $q = 34$, $m\angle Q = 111$

11. If $m\angle P = 89$, $p = 16$, $r = 12$

12. If $m\angle Q = 103$, $m\angle P = 63$, $p = 13$

13. If $m\angle P = 96$, $m\angle R = 82$, $r = 35$

14. If $m\angle R = 49$, $m\angle Q = 76$, $r = 26$

15. If $m\angle Q = 31$, $m\angle P = 52$, $p = 20$

16. If $q = 8$, $m\angle Q = 28$, $m\angle R = 72$

17. If $r = 15$, $p = 21$, $m\angle P = 128$

1.

$180 - 66 - 82$
$=$
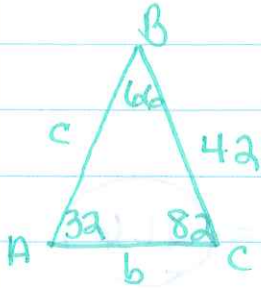
$\dfrac{\sin 32}{42} = \dfrac{\sin 66}{b}$

$b = \dfrac{42 \sin 66}{\sin 32}$
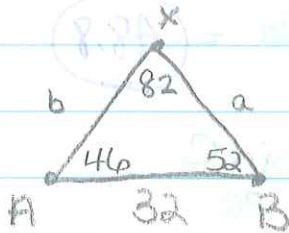
$b = \boxed{72.4}$

$\dfrac{\sin 32}{42} = \dfrac{\sin 82}{c}$

$c = \dfrac{42 \sin 82}{\sin 32}$

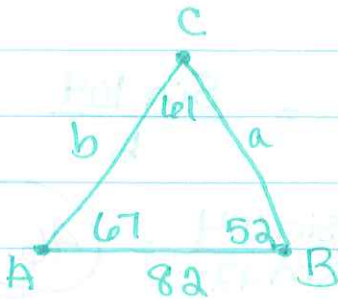$c = \boxed{78.5}$

2.

$180 - 46 - 52$

$\angle X = 82$

$\dfrac{\sin 46}{a} = \dfrac{\sin 82}{32}$

$a = \dfrac{32 \sin 46}{\sin 82} = \boxed{23.3}$

$\dfrac{\sin 52}{b} = \dfrac{\sin 82}{32}$

$b = \dfrac{32 \sin 52}{\sin 82} = \boxed{25.5}$

3.

$180 - 67 - 52 =$

$\angle C = 61$

$\dfrac{\sin 61}{82} = \dfrac{\sin 67}{a}$

$a = \dfrac{82 \sin 67}{\sin 61} = \boxed{86.3}$

$\dfrac{\sin 61}{82} = \dfrac{\sin 52}{b}$

$b = \dfrac{82 \sin 52}{\sin 61} = \boxed{73.9}$
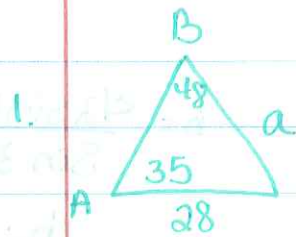
4.

$180 - 68 - 43$

$\angle C = 69$
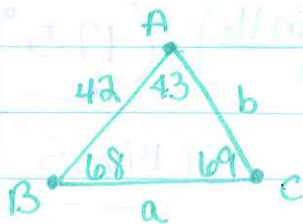
$\dfrac{\sin 69}{42} = \dfrac{\sin 43}{a}$

$a = \dfrac{42 \sin 43}{\sin 69} = \boxed{30.7}$

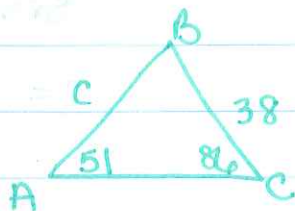$\dfrac{\sin 69}{42} = \dfrac{\sin 68}{b}$

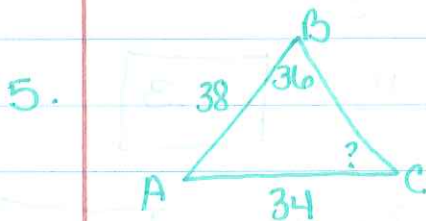$b = \dfrac{42 \sin 68}{\sin 69} = \boxed{41.7}$

1.

$$\frac{\sin 35}{a} = \frac{\sin 48}{28}$$

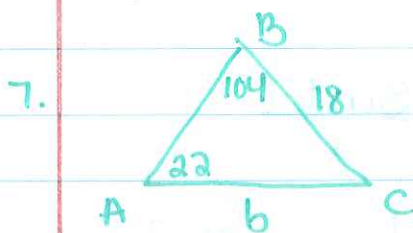$$a = \frac{28 \sin 35}{\sin 48} = \boxed{21.6}$$

3.

$$\frac{\sin 51}{38} = \frac{\sin 86}{c}$$

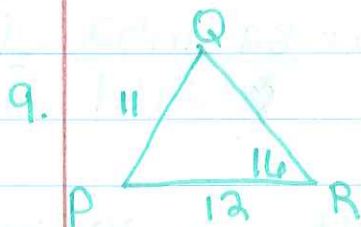$$c = \frac{38 \sin 86}{\sin 51} = \boxed{48.8}$$

5.

$$\frac{\sin 36}{34} = \frac{\sin C}{38}$$

$$C = \sin^{-1}\left(\frac{38 \sin 36}{34}\right) = \boxed{41.1^\circ}$$

7.

$$\frac{\sin 22}{18} = \frac{\sin 104}{b}$$

$$b = \frac{18 \sin 104}{\sin 22} = \boxed{46.4}$$
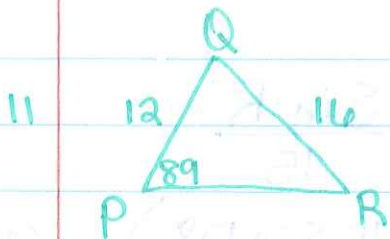
9.

$$\frac{\sin Q}{12} = \frac{\sin 16}{11} = \frac{\sin P}{P}$$

$$Q = \sin^{-1}\left(\frac{12 \sin 16}{11}\right) = \boxed{17.5^\circ}$$

$$\angle P = 180 - 16 - 17.5 = \boxed{146.5^\circ}$$

$$\frac{\sin 16}{11} = \frac{\sin 146.5}{P}$$

$$P = \frac{11 \sin 146.5}{\sin 16} = \boxed{22.0}$$
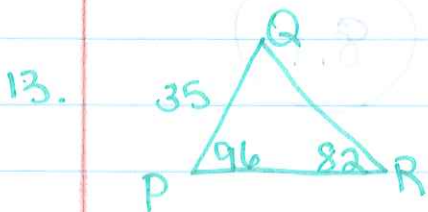
11.

$$\frac{\sin 89}{16} = \frac{\sin R}{12}$$

$$R = \sin^{-1}\left(\frac{12 \sin 89}{16}\right) = \boxed{48.4}$$

$$\angle Q = 180 - 89 - 48.6 = 42.4$$

$$\frac{\sin 89}{16} = \frac{\sin 42.4}{q}$$

$$q = \frac{16 \sin 42.4}{\sin 89} = \boxed{10.8}$$
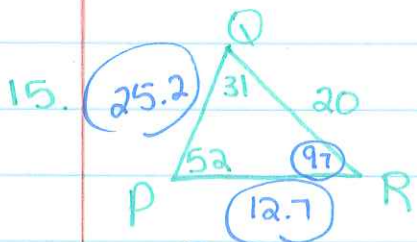
13.

$$\frac{\sin 82}{35} = \frac{\sin 96}{P}$$

$$P = \frac{35 \sin 96}{\sin 82} = \boxed{35.2}$$

$$\angle Q = 180 - 96 - 82 = 2$$

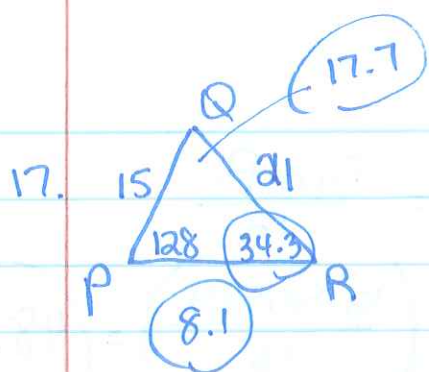$$\frac{\sin 82}{35} = \frac{\sin 2}{q} \qquad q = \frac{35 \sin 2}{\sin 82} = \boxed{1.2}$$

15. $\boxed{25.2}$ $\boxed{97}$ $\boxed{12.7}$

$$\angle R = 180 - 52 - 31 = \boxed{97}$$

$$\frac{\sin 52}{20} = \frac{\sin 30}{q}$$

$$q = \frac{20 \sin 30}{\sin 52} = \boxed{12.7}$$

$$\frac{\sin 97}{r} = \frac{\sin 52}{20}$$

$$r = \frac{20 \sin 97}{\sin 52} = \boxed{25.2}$$

17.

Q (17.7)

15   21

128 (34.3)

P   R

(8.1)

$$\frac{\sin 128}{21} = \frac{\sin R}{15}$$

$$R = \sin^{-1}\left(\frac{15 \sin 128}{21}\right) = (34.3°)$$

$$180 - 128 - 34.3 = 17.7°$$

$$\frac{\sin 17.7}{q} = \frac{\sin 128}{21}$$

$$q = \frac{21 \sin 17.7}{\sin 128} = (8.1)$$