**7-4 Double-Angle and Half-Angle Identities** **Advanced Math**

## Warm-Up

Use the sum or difference identity to find the exact value of sin 375°

$\sin 375 = \sin(330 + 45)$

$= \sin 330 \cos 45 + \cos 330 \sin 45$

$= -\frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= -\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$

1. If $\sin \theta = \frac{3}{4}$ and $\theta$ has its terminal side in the first quadrant, find the exact value of each function.

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{3}{4}\right)^2 + \cos^2\theta = 1$

$\cos^2\theta = \frac{7}{16}$

$\cos\theta = \frac{\sqrt{7}}{4}$

$\tan\theta = \frac{\frac{3}{4}}{\frac{\sqrt{7}}{4}} = \frac{3}{\cancel{4}} \cdot \frac{\cancel{4}}{\sqrt{7}} = \frac{3}{\sqrt{7}}$

**Double-Angle Identities**

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

a. $\sin 2\theta$

$2\sin\theta\cos\theta$

$2\left(\frac{3}{4}\right)\left(\frac{\sqrt{7}}{4}\right)$

$\frac{6\sqrt{7}}{16}$

$\frac{3\sqrt{7}}{8}$

b. $\cos 2\theta$

$\cos^2\theta - \sin^2\theta$

$\left(\frac{\sqrt{7}}{4}\right)^2 - \left(\frac{3}{4}\right)^2$

$\frac{7}{16} - \frac{9}{16}$

$\frac{-2}{16} = \frac{-1}{8}$

c. $\tan 2\theta$

$\frac{2\tan\theta}{1 - \tan^2\theta}$

$\frac{2\left(\frac{3}{\sqrt{7}}\right)}{1 - \left(\frac{3}{\sqrt{7}}\right)^2} = \frac{\frac{6}{\sqrt{7}}}{1 - \frac{9}{7}}$

$= \frac{\frac{6}{\sqrt{7}}}{\frac{-2}{7}} = \frac{\cancel{6}^{3}}{\sqrt{7}} \cdot \frac{7}{\cancel{-2}} = \frac{-21}{\sqrt{7}}$

$= \frac{-21}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{-21\sqrt{7}}{7} = -3\sqrt{7}$

2. Use a half-angle identity to find the exact value of each function.

**Half-Angle Identities**

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} \qquad \cos\alpha \neq -1$$

a. $\cos\frac{\pi}{8} = \cos\frac{\frac{\pi}{4}}{2}$ since in Quad I, use + value

$$= \pm\sqrt{\frac{1+\cos\frac{\pi}{4}}{2}} = \pm\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \pm\sqrt{\frac{2+\sqrt{2}}{4}} = \pm\frac{\sqrt{2+\sqrt{2}}}{2}$$

b. $\tan 15° = \tan\frac{30}{2}$

$$= \pm\sqrt{\frac{1-\cos 30}{1+\cos 30}} = \sqrt{\frac{1-\sqrt{3}/2}{1+\sqrt{3}/2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}} \cdot \frac{\sqrt{2-\sqrt{3}}}{\sqrt{2-\sqrt{3}}}$$

$$= \frac{\sqrt{(2-\sqrt{3})^2}}{\sqrt{4-3}} = \frac{2-\sqrt{3}}{\sqrt{1}} = 2-\sqrt{3}$$

3. Verify that $1 - \cos 2x \sec^2 x = \tan^2 x$ is an identity

$$1 - \frac{\cos^2 x - \sin^2 x}{\cos^2 x} = \tan^2 x$$

$$1 - \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = \tan^2 x$$

$$\cancel{1} - \cancel{1} + \frac{\sin^2 x}{\cos^2 x} = \tan^2 x$$

$$\tan^2 x = \tan^2 x$$