Warm-Up: Verify that 1 = tan x cos x csc x is an identity.

$$1 = \frac{\cancel{\sin x}}{\cancel{\cos x}} \cdot \cancel{\cos x} \cdot \frac{1}{\cancel{\sin x}}$$

$$1 = 1$$

I. Sum and Difference Identities

Sum and Difference Identities for the Cosine, Sine, and Tangent Functions:

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

1a. Show by producing a counterexample that
$\cos(x - y) \neq \cos x - \cos y$

$$\cos(\pi - \pi) \neq \cos \pi - \cos \pi$$

$$\cos(0) \neq -1 - {}^{-}1$$

$$1 \neq 0$$

b. Show that the difference identity for cosine is true for the values used in part a.

$$\cos(\pi - \pi) = \cos\pi \cos\pi + \sin\pi \sin\pi$$

$$\cos(0) = -1 \cdot -1 + 0 \cdot 0$$

$$1 = 1$$

2. Use the sum or difference identity for cosine to find the exact value of cos 75°

* use 2 angles that add or subtract to get this

$$\cos(30+45) = \cos 30 \cos 45 - \sin 30 \sin 45$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$$

3. Use the sum or difference identity for cosine to find the exact value of cos 735°

$735-360-360=15°$

$\cos 15 = \cos(45-30) = \cos 45 \cos 30 + \sin 45 \sin 30$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$= \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

4. Find the value of sin (x + y) if $0 < x < \frac{\pi}{2}$, $0 < y < \frac{\pi}{2}$, $\sin x = \frac{4}{5}$ and $\sin y = \frac{5}{13}$

$\sin x \cos y + \sin y \cos x$

$\frac{4}{5}\left(\frac{12}{13}\right) + \frac{5}{13}\left(\frac{3}{5}\right)$

$\frac{48}{65} + \frac{15}{65} = \boxed{\frac{63}{65}}$

$\left(\frac{4}{5}\right)^2 + \cos^2 x = 1$

$\cos^2 x = \frac{9}{25}$

$\cos x = \frac{3}{5}$

$\left(\frac{5}{13}\right)^2 + \cos^2 y = 1$

$\cos^2 y = \frac{144}{169}$

$\cos y = \frac{12}{13}$

5. Use the sum or difference identity for tangent to find the exact value of tan 255°

$\tan 255 = \tan(225+30) = \frac{\tan 225 + \tan 30}{1 - \tan 225 \tan 30}$

$= \frac{1+\frac{\sqrt{3}}{3}}{1-\frac{\sqrt{3}}{3}} \cdot \frac{3}{3}$

$= \frac{3+\sqrt{3}}{3-\sqrt{3}} \cdot \frac{3+\sqrt{3}}{3+\sqrt{3}} = \frac{9+6\sqrt{3}+3}{9-3} = \frac{12+6\sqrt{3}}{6}$

$= \boxed{2+\sqrt{3}}$

6. Verify that sec (π + A) = - sec A is an identity

$\frac{1}{\cos(\pi+A)} = -\sec A$

$\frac{1}{\cos\pi \cos A - \sin\pi \sin A} = -\sec A$

$-\sec A = -\sec A$

$\frac{1}{-1 \cdot \cos A - \cancel{0 \cdot \sin A}} = -\sec A$

$\frac{1}{-\cos A} = -\sec A$