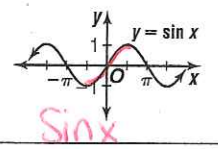
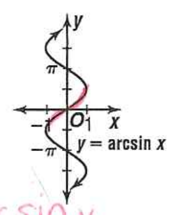
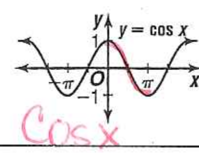
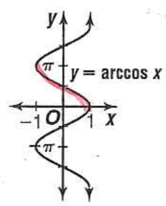
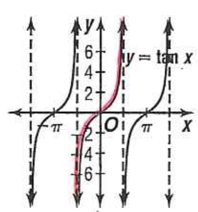
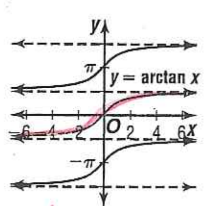
**6-8 Trigonometric Inverses and Their Graphs** **Advanced Math**

An **inverse** of a function may be found by interchanging the coordinates of the ordered pairs of a function

The **domain** of the function becomes the Range

The **range** of the function becomes the Domain

Is an inverse of a function always a function? NO

| Relation | Ordered Pairs | Graph | Domain | Range |
|---|---|---|---|---|
| $y = \sin x$ | $(x, \sin x)$ |   Sinx | All real numbers $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ | $-1 \le y \le 1$ |
| $y = \arcsin x$ | $(\sin x, x)$ |   Arcsinx | $-1 \le x \le 1$ | All real numbers $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ |
| $y = \cos x$ | $(x, \cos x)$ |   Cosx | All real numbers $0 \le x \le \pi$ | $-1 \le y \le 1$ |
| $y = \text{arcos } x$ | $(\cos x, x)$ |   | $-1 \le x \le 1$ | All real numbers $0 \le y \le \pi$ |
| $y = \tan x$ | $(x, \tan x)$ |   Tanx | All real numbers except $\frac{\pi}{2}n$, where $n$ is an odd integer $-\frac{\pi}{2} < x < \frac{\pi}{2}$ | All real numbers |
| $y = \arctan x$ | $(\tan x, x)$ |   Arctan | All real numbers | All real numbers except $\frac{\pi}{2}n$, where $n$ is an odd integer $-\frac{\pi}{2} < y < \frac{\pi}{2}$ |

Are any of the inverses above functions? NO

## 6-8 Trigonometric Inverses and Their Graphs 

We can make these functions by considering only a part of the domain.

right side unit circle

top ½ unit circle

right side unit circle

| Function | Domain | Range |
|---|---|---|
| $y = \text{Sin } x$ | $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ | $-1 \le y \le 1$ |
| $y = \text{Arcsin } x$ | $-1 \le x \le 1$ | $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ |
| $y = \text{Cos } x$ | $0 \le x \le \pi$ | $-1 \le y \le 1$ |
| $y = \text{Arccos } x$ | $-1 \le x \le 1$ | $0 \le y \le \pi$ |
| $y = \text{Tan } x$ | $-\frac{\pi}{2} < x < \frac{\pi}{2}$ | all real numbers |
| $y = \text{Arctan}$ | all real numbers | $-\frac{\pi}{2} < y < \frac{\pi}{2}$ |

The values in the domain are called principal values. (These make it a function)

★ show red on other side

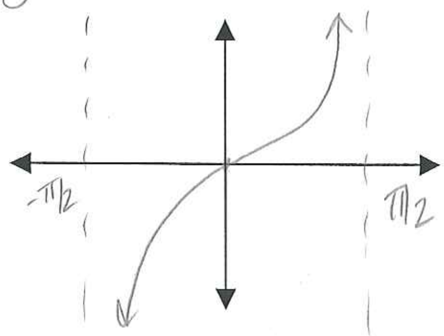
**Ex:** Write the equation for the inverse of $y = \text{Arctan } \frac{1}{2}x$. Then graph the function and its inverse

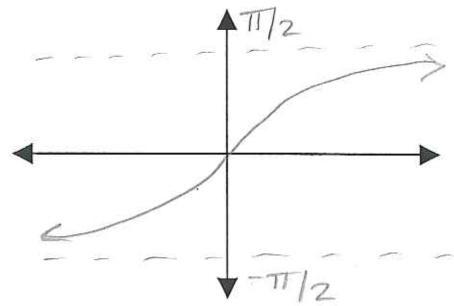
$y = 2\text{Tan}x$

$X = \text{Arctan}\frac{1}{2}Y$

$\text{Tan } x = \frac{1}{2}y$

$2\text{Tan } x = y$




**Ex: Find each value.** ★ Look at Domain & Range values

a. $\text{Arcsin } -\frac{1}{2}$ What angle has sine $-\frac{1}{2}$?

$\sin\theta = -\frac{1}{2}$

$\theta = \boxed{-\pi/6}$

b. $\text{Tan}^{-1} (\sin \frac{\pi}{2})$

$\text{Tan}^{-1}(1)$

$\boxed{\pi/4}$

c. $\cos (\text{Arctan } \sqrt{3} - \text{Arcsin } \frac{\sqrt{3}}{2})$

$\tan\theta = \sqrt{3}$

$\sin\theta = \frac{\sqrt{3}}{2}$

$\cos\left(\frac{\pi}{3} - \frac{\pi}{3}\right)$

$\cos(0) = \boxed{1}$

d. $\tan [\text{Tan}^{-1}(\sqrt{3}) + \frac{\pi}{3}]$

$\tan\theta = \sqrt{3}$

$\tan\left[\frac{\pi}{3} + \frac{\pi}{3}\right]$

$\tan\left[\frac{2\pi}{3}\right]$

$\boxed{-\sqrt{3}}$

**Ex:** Determine whether $\text{Sin}^{-1} (\sin x) = x$ is true or false for all values of x. If false, give a counterexample.

When $x = \frac{2\pi}{3}$ $\quad \sin x = \frac{\sqrt{3}}{2}$

$\text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3} \neq x$

False