

12-5

Practice

Sigma Notation and the n th Term

Write each expression in expanded form and then find the sum.

1. $\sum_{n=3}^5 (n^2 - 2^n)$

$(3^2 - 2^3) + (4^2 - 2^4) + (5^2 - 2^5) = -6$

2. $\sum_{q=1}^4 \frac{2}{q} = \frac{2}{1} + \frac{2}{2} + \frac{2}{3} + \frac{2}{4} = \frac{25}{6}$

3. $\sum_{t=1}^5 t(t-1)$

$1(1-1) + 2(2-1) + 3(3-1) + 4(4-1) + 5(5-1) = 40$

4. $\sum_{t=0}^3 (2t - 3)$

$(2(0)-3) + (2(1)-3) + (2(2)-3) + (2(3)-3) = -3 + -1 + 1 + 3 = 0$

5. $\sum_{c=2}^5 (c-2)^2$

$(1-2)^2 + (3-2)^2 + (4-2)^2 + (5-2)^2 = 14$

6. $\sum_{i=1}^{\infty} 10\left(\frac{1}{2}\right)^i$ $r = \frac{1}{2} < 1 \checkmark$

$5 + \frac{5}{2} + \frac{5}{4} + \frac{5}{8} + \dots$
 $\frac{5}{1 - \frac{1}{2}} = 10$

Express each series using sigma notation.

7. $3 + 6 + 9 + 12 + 15$

$\sum_{n=1}^5 3n$

8. $6 + 24 + 120 + \dots + 40,320$

$\sum_{n=3}^8 n!$

9. $\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{100}$

$\sum_{n=1}^{10} \frac{1}{n^2}$

10. $24 + 19 + 14 + \dots + (-1)$

$\sum_{n=0}^5 (24 - 5n)$

11. **Savings** Kathryn started saving quarters in a jar. She began by putting two quarters in the jar the first day and then she increased the number of quarters she put in the jar by one additional quarter each successive day.

$2 + 3 + 4 + 5$

- a. Use sigma notation to represent the total number of quarters Kathryn had after 30 days.

$\sum_{n=1}^{30} (n+1)$

- b. Find the sum represented in part a.

$\frac{30}{2} (2(2) + 1(30-1)) = 495$

Express each series using sigma notation.

12. $5 + 10 + 15 + 20 + 25$

$$\sum_{n=1}^5 5n$$

13. $2 + 4 + 10 + 28$

$$\sum_{n=0}^3 (3^n + 1)$$

14. $2 - 4 - 10 - 16$

$$\sum_{n=1}^4 (8 - 6n)$$

16. $\frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \frac{3}{32} + \dots$

$$\sum_{n=2}^{\infty} \frac{3}{2^n}$$

17. $5 + 5 + \frac{5}{2} + \frac{5}{6} + \frac{5}{24} + \dots$

$$\sum_{n=0}^{\infty} \frac{5}{n!}$$

18. $\frac{3}{9 \cdot 2} + \frac{8}{27 \cdot 6} + \frac{15}{81 \cdot 24} + \dots$

$$\sum_{n=1}^{\infty} \frac{n^2 - 1}{3^n n!}$$

19. $\frac{1 \cdot 2}{2} + \frac{2 \cdot 3}{4} + \frac{3 \cdot 4}{6} + \dots + \frac{8 \cdot 9}{16}$

$$\sum_{n=1}^8 \frac{n \cdot (n+1)}{2n}$$