12-5 Sigma Notation Practice

Name: Key

Express each series using sigma notation.

1. $3 - 5 + 7 - 9 + 11 - 13 + 15$

$$\sum_{n=0}^{6} (3+2n)(-1)^n$$

2. $\frac{1}{2} + \frac{1}{5} + \frac{1}{8} + \frac{1}{11} + \frac{1}{14} + \frac{1}{17}$

$$\sum_{n=0}^{5} \frac{1}{2+3n}$$

3. $1 + 3 + 7 + 15 + 31 + \ldots$

$$\sum_{n=1}^{\infty} 2^n - 1$$

4. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \frac{6}{7}$

$$\sum_{n=1}^{6} \frac{n}{n+1}$$

5. $-2 + 4 - 8 + 16 - \ldots + 1024$

$$\sum_{n=1}^{10} (-2)^n$$

6. $\frac{2}{2} - \frac{4}{6} + \frac{6}{24} - \frac{8}{120} + \frac{10}{720}$

$$\sum_{n=2}^{6} \frac{(-1)^n(2n-2)}{n!}$$

7. $5 + \frac{15}{4} + \frac{45}{16} + \frac{135}{64} + \cdots$

$$\sum_{n=0}^{\infty} \frac{5 \cdot 3^n}{4^n}$$

8. $6 + 12 + 20 + 30 + 42$

$$\sum_{n=2}^{6} (n^2+n)$$

9. $13 + 9 + 5 + 1 + \ldots$

$$\sum_{n=0}^{\infty} (13-4n)$$

10. $-1 + 1 + 3 + 5 + \ldots$

$$\sum_{n=0}^{\infty} (-1+2n)$$

11. $\frac{\sqrt{2}}{3} + \frac{2}{6} + \frac{\sqrt{8}}{18} + \frac{4}{72} + \frac{\sqrt{32}}{360} + \cdots$

$$\sum_{n=1}^{\infty} \frac{\sqrt{2}^n}{3n!}$$

12. $(2 \cdot 3) + (4 \cdot 5) + (6 \cdot 7) + \ldots$

$$\sum_{n=1}^{\infty} 2n \cdot (2n+1)$$

13. $\frac{1}{4} + 0 + 4 + \cdots + 65{,}536$

$$\sum_{n=0}^{9} 4^{n-1}$$

14. $\frac{2 \cdot 4}{3} + \frac{3 \cdot 8}{9} + \frac{4 \cdot 16}{27} + \cdots + \frac{8 \cdot 256}{2187}$

$$\sum_{n=1}^{7} \frac{(n+1)2^{n+1}}{3^n}$$

15. $3+\frac{6}{2}+\frac{9}{6}+\frac{12}{24}+\frac{15}{120}$

$\sum_{n=1}^{5} \frac{3n}{n!}$

16. $4-8-20-32 \ldots -80$

$-12 \quad -12$

$\sum_{n=0}^{7} 4-12n$

17. $\frac{2}{2\cdot 6}+\frac{5}{4\cdot 24}+\frac{10}{8\cdot 120}+\frac{17}{16\cdot 720}+\cdots+\frac{50}{128\cdot 362880}$

$+3 \quad +5 \quad +7$

$\sum_{n=1}^{7} \frac{n^2+1}{2^n(n+2)!}$

18. Each October Albuquerque, New Mexico, hosts the Balloon Fiesta. In 2013, 744 hot air balloons participated in the opening day festivities. One of these balloons rose 389 feet after 1 minute. Because the air in the balloon was not reheated, each succeeding minute the balloon rose 63% as far as it did the previous minute.

a. Use sigma notation to represent the height of the balloon above the ground after one hour. Then calculate the total height of the balloon after one hour to the nearest foot.

$\sum_{0}^{59} 389(0.63)^n$ or $\sum_{n=1}^{60} 389(0.63)^{n-1}$

$S=\frac{389}{1-0.63}$

≈ 1051.4 ft

b. What was the maximum height achieved by this balloon.

1051.4 ft

19. A popular shoe manufacturer is planning to market a new style of tennis shoe in a city of 500,000 people. Using a prominent professional athlete as their spokesperson, the company's ad agency hopes to induce 35% of the people to buy the product. The ad agency estimates that these satisfied customers will convince 35% of 35% of 500,000 to buy a pair of shoes, and those will persuade 35% of 35% of 35% of 500,000, and so on.

a. Model this situation using sigma notation.

$\sum_{n=1}^{\infty} 500{,}000(0.35)^n$

b. Find the total number of people that will buy the product as a result of the advertising campaign.

$500{,}000(0.35)^1 = 175000$

$S=\frac{175000}{1-0.35}=$ 269,230 people

c. What percentage of the population is this?

$\frac{269230}{500000} \approx 53.8\%$

d. What important assumption does the advertising agency make in proposing the figure found in part **b** to the shoe manufacturer?

The ad agency assumes that the people who buy the shoes will be satisfied with their purchase