12-3

NAME Key DATE ________ PERIOD ________

Practice

Infinite Sequence and Series

Find each limit, or state that the limit does not exist and explain your reasoning.

1. $\lim_{n\to\infty} \frac{n^2-1}{n^2+1}$ $\frac{n^2/n^2 - 1/n^2}{n^2/n^2 + 1/n^2} = \frac{1-\frac{1}{n^2}}{1+\frac{1}{n^2}}$

2. $\lim_{n\to\infty} \frac{4n^2-5n}{3n^2+4}$

3. $\lim_{n\to\infty} \frac{5n^2+1}{6n}$

4. $\lim_{n\to\infty} \frac{(n-1)(3n+1)}{5n^2}$

5. $\lim_{n\to\infty} \frac{3n-(-1)^n}{4n^2}$

6. $\lim_{n\to\infty} \frac{n^3+1}{n^2}$

Write each repeating decimal as a fraction.

7. $0.\overline{75}$

8. $0.\overline{592}$

Find the sum of each infinite series, or state that the sum does not exist and explain your reasoning.

9. $\frac{2}{5} + \frac{6}{25} + \frac{18}{125} + \cdots$

10. $\frac{3}{4} + \frac{15}{8} + \frac{75}{16} + \cdots$

11. *Physics* A tennis ball is dropped from a height of 55 feet and bounces $\frac{3}{5}$ of the distance after each fall.
   a. Find the first seven terms of the infinite series representing the vertical distances traveled by the ball.
   b. What is the total vertical distance the ball travels before coming to rest?

Find each limit, or state that the limit does not exist.

1. $\lim_{n \to \infty} \frac{3n^4}{2n^2 + 5}$ 2. $\lim_{n \to \infty} \frac{(2n + 1)(n - 2)}{2n^2}$ 3. $\lim_{n \to \infty} \frac{n + 1}{n^2 - 4}$

Find the sum of each series, or state that the sum does not exist.

4. $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \cdots$ 5. $-\frac{3}{5} + 1 - \frac{5}{3} + \cdots$

6. Find $\lim_{n \to \infty} \frac{n^2 + 1}{2n^2 - 3n}$ or state that the limit does not exist.

7. Find the sum of the series $\frac{1}{8} + \frac{1}{4} + \frac{1}{2} + \cdots$ or state that the sum does not exist.

Find each limit, or state that the limit does not exist and explain your reasoning.

8. $\lim_{n\to\infty} \frac{3n}{4n+1}$ 9. $\lim_{n\to\infty} \frac{6n-3}{n}$ 10. $\lim_{n\to\infty} \frac{2^n n^3}{3n^3}$

11. $\lim_{n\to\infty} \frac{4n^3-3n}{n^4-4n^3}$ 12. $\lim_{n\to\infty} \frac{n^3-n^2+4}{5+2n^3}$ 13. $\lim_{n\to\infty} \frac{n^4-3n}{n^3}$

Find the sum of the series, or state that the sum does not exist and explain your reasoning.

14. $\frac{2}{7} + \frac{4}{7} + \frac{8}{7} + \cdots$ 15. 1260 + 504 + 201.6 + 80.64 + ...

12.3 Practice #1

① $\lim_{n\to\infty} \frac{n^2-1}{n^2+1} = \boxed{1}$

$\lim_{n\to\infty} \frac{\cancel{n^2}/\cancel{n^2} - 1/n^2}{\cancel{n^2}/\cancel{n^2} + 1/n^2} = \frac{1-0}{1+0} = \boxed{1}$

② $\lim_{n\to\infty} \frac{4n^2-5n}{3n^2+4} =$

$\lim_{n\to\infty} \frac{\frac{4\cancel{n^2}}{\cancel{n^2}} - \frac{5\cancel{n}}{n^2}}{\frac{3\cancel{n^2}}{\cancel{n^2}} + \frac{4}{n^2}} = \frac{4-0}{3+0} = \boxed{\frac{4}{3}}$

③ $\lim_{n\to\infty} \frac{5n^2+1}{6n}$

$\lim_{n\to\infty} \frac{5n^{\cancel{2}}}{6\cancel{n}} + \frac{1}{6n} = \frac{5}{6}n + \frac{1}{6n}$

$\uparrow$ DNE $+0$

$\boxed{\text{no limit}}$

OR

$\frac{\frac{5n^2}{n^2} + \frac{1}{n^2}}{\frac{6n}{n^2}} = \frac{5+0}{0}$

Can't divide by 0 so DNE.

④ $\lim_{n\to\infty} \frac{(n-1)(3n+1)}{5n^2} = \frac{3n^2-2n-1}{5n^2} = \frac{\frac{3n^2}{n^2} - \frac{2n}{n^2} - \frac{1}{n^2}}{\frac{5n^2}{n^2}} = \frac{3-0-0}{5}$

$= 3/5$

⑤ $\lim_{n\to\infty} \frac{3n-(-1)^n}{4n^2} = \frac{3n}{4n^2} - \frac{(-1)^n}{4n^2} = \frac{3}{4n} - \frac{\pm 1}{4n^2} = 0 - 0 = 0$

⑥ $\lim_{n\to\infty} \frac{n^3+1}{n^2} = \frac{n^3}{n^2} + \frac{1}{n^2} = n + \frac{1}{n^2} =$ DNE $+0$

no limit

⑦ $0.\overline{75} = \frac{75}{100} + \frac{75}{10000} + \frac{75}{100000} \ldots$

$r = \frac{1}{100}$

$S = \frac{\frac{75}{100}}{1-\frac{1}{100}} = \frac{\frac{75}{100}}{\frac{99}{100}} = \frac{75}{99}$

$= \frac{25}{33}$

⑧ $0.\overline{592} = \frac{592}{1{,}000} + \frac{592}{1{,}000{,}000} + \frac{592}{1{,}000{,}000{,}000} + \ldots$

$r = \frac{1}{1000}$

$S = \frac{\frac{592}{1000}}{1-\frac{1}{1000}} = \frac{\frac{592}{1000}}{\frac{999}{1000}} = \frac{592}{999} = \frac{16}{27}$

(9) $\frac{2}{5}+\frac{6}{25}+\frac{18}{125}+\dots$

$r=\dfrac{\frac{6}{25}}{\frac{2}{5}}=\frac{\cancel{6}^{3}}{\cancel{25}_{5}}\cdot\frac{\cancel{5}^{1}}{\cancel{2}_{1}}=\frac{3}{5}$

$S=\dfrac{\frac{2}{5}}{1-\frac{3}{5}}=\dfrac{\frac{2}{5}}{\frac{2}{5}}=1$

(10) $\frac{3}{4}+\frac{15}{8}+\frac{75}{16}+\dots$ $r=\frac{15}{8}\div\frac{3}{4}=\frac{\cancel{15}^{5}}{{}_{2}\cancel{8}}\cdot\frac{\cancel{4}^{1}}{\cancel{3}_{1}}=5/2$

Sum doesn't exist b/c $5/2>1$.

(11)

a. $55+33+33+\underset{19.8}{19\tfrac{4}{5}}+\underset{19.8}{19\tfrac{4}{5}}+11.88+11.88\dots$

$55(3/5)=33$

$33(3/5)=19.8$

b. $55+2\left(\frac{33}{1-3/5}\right)=$ 220 feet.

Back of worksheet:

1. $\lim_{n\to\infty}\frac{3n^4}{2n^2+5}=\dfrac{\frac{3n^4}{n^4}}{\frac{2n^2}{n^4}+\frac{5}{n^4}}=\dfrac{3}{\frac{2}{n^2}+\frac{5}{n^4}}=\frac{3}{0}$ no limit

2. $\lim_{n\to\infty}\frac{(2n+1)(n-2)}{2n^2}=\frac{2n^2-3n-2}{2n^2}=\frac{2n^2}{2n^2}-\frac{3n}{2n^2}-\frac{2}{2n^2}$

$=1-\frac{3}{2n}-\frac{1}{n^2}=1-0-0=1$

3. $\lim_{n\to\infty}\frac{n+1}{n^2-4}=\dfrac{\frac{n}{n^2}+\frac{1}{n^2}}{\frac{n^2}{n^2}-\frac{4}{n^2}}=\dfrac{\frac{1}{n}+\frac{1}{n^2}}{1-\frac{4}{n^2}}=\frac{0+0}{1-0}=\frac{0}{1}=0$

4. $\frac{1}{2}-\frac{1}{4}+\frac{1}{8}-\frac{1}{16}+\dots$

$r=\frac{-1}{4}\div\frac{1}{2}=\frac{-1}{4}\cdot\frac{2}{1}=\frac{-1}{2}$

$S=\dfrac{\frac{1}{2}}{1+1/2}=\dfrac{\frac{1}{2}}{\frac{3}{2}}=\frac{1}{2}\cdot\frac{2}{3}=\frac{1}{3}$

5. $-\frac{3}{5}+1-\frac{5}{3}+\dots$ $r=\frac{1}{-3/5}=-5/3$

$S=\dfrac{-3/5}{1+5/3}=\dfrac{-3/5}{8/3}=\frac{-3}{5}\cdot\frac{3}{8}=\frac{-9}{40}$

6. $\lim_{n\to\infty} \frac{n^2+1}{2n^2-3n} = \frac{\frac{n^2}{n^2}+\frac{1}{n^2}}{\frac{2n^2}{n^2}-\frac{3n}{n^2}} = \frac{1+0}{2-0} = \frac{1}{2}$

7. $\frac{1}{8}+\frac{1}{4}+\frac{1}{2}+\ldots$ $\quad r = \frac{1}{4} \div \frac{1}{8} = \frac{1}{4}\cdot\frac{8}{1} = 2$

$S = \frac{\frac{1}{8}}{1-2} = \frac{1}{8}\cdot(-1) = -\frac{1}{8}$

8. $\lim_{n\to\infty} \frac{3n}{4n+1} = \frac{3n/n}{4n/n+1/n} = \frac{3}{4+0} = 3/4$

9. $\lim_{n\to\infty} \frac{6n-3}{n} = \frac{6n/n - 3/n}{n/n} = \frac{6-0}{1} = 6$

10. $\lim_{n\to\infty} \frac{2^n n^3}{3n^3} = \lim_{n\to\infty} 2^n \cdot \lim_{n\to\infty} \frac{n^3/n^3}{3n^3/n^3} = \text{DNE}\cdot\frac{1}{3} =$ no limit

11. $\lim_{n\to\infty} \frac{4n^3-3n}{n^4-4n^3} = \frac{4n^3/n^4 - 3n/n^4}{n^4/n^4 - 4n^3/n^4} = \frac{4/n - 3/n^3}{1-4/n} = \frac{0}{1} = 0$

12. $\lim_{n\to\infty} \frac{n^3-n^2+4}{5+2n^3} = \frac{n^3/n^3 - n^2/n^3 + 4/n^3}{5/n^3 + 2n^3/n^3} = \frac{1-\frac{1}{n}+\frac{4}{n^3}}{\frac{5}{n^3}+2} = \frac{1}{2}$

13. $\lim_{n\to\infty} \frac{n^4-3n}{n^3} = \frac{n^4/n^4 - 3n/n^4}{n^3/n^4} = \frac{1-3/n^3}{1/n} = \frac{1}{0}$ no limit

14. $\frac{2}{7}+\frac{4}{7}+\frac{8}{7}+\ldots$

$\frac{4}{7}\div\frac{2}{7} = \frac{4}{7}\cdot\frac{7}{2} = 2$

no sum exists b/c $2>1$

15. $1260+504+201.6+80.64+\ldots$

$r = \frac{504}{1260} = 0.4$ $\quad S = \frac{1260}{1-0.4} = 2100$