14 – 30

1d. $\lim_{n\to\infty} \frac{n-1}{n} = \lim_{n\to\infty} \frac{n}{n} - \lim_{n\to\infty} \frac{1}{n}$
$= 1 - 0$
$= 1$

The limits are equal.

2a. See students' work. Student's should draw the following conclusions:

$\lim_{n\to\infty}\left(\frac{1}{2}\right)^n = 0$

$\lim_{n\to\infty}\left(\frac{1}{4}\right)^n = 0$

$\lim_{n\to\infty}(1)^n = 1$

$\lim_{n\to\infty}(2)^n =$ no limit

$\lim_{n\to\infty}(5)^n =$ no limit

A#12.3
p781
13-23 all
30-40 even

2b. If $|r| < 1$, then $\lim_{n\to\infty} r^n = 0$. If $|r| =$. $\lim_{n\to\infty} r^n = 1$. If $|r| > 1$ then $\lim_{n\to\infty} r^n$ does not exist.

3. Sample answer: $2 + 4 + 8 + \cdots$

4. Zonta is correct. As n approaches infinity the expression $2n - 3$ will continue to grow larger and larger. Tyree applied the method of dividing by the highest powered term incorrectly. Both the numerator and the denominator of the expression must be divided by the highest-powered term. It is not appropriate to apply this method here since the denominator of the expression $2n - 3$ is 1.

5. 0; as $n \to \infty$, 5^n becomes increasingly large and thus the value $\frac{1}{5^n}$ becomes smaller and smaller, approaching zero. So the sequence has a limit of zero.

6. $\lim_{n\to\infty} \frac{5 - n^2}{2n} = \lim_{n\to\infty}\left(\frac{5}{2n} - \frac{1}{2}n\right)$

$\lim_{n\to\infty} \frac{5}{2n} = \lim_{n\to\infty} \frac{5}{2} \cdot \frac{1}{n} = \frac{5}{2} \cdot 0 = 0$

As n approaches infinity, $\frac{1}{2}n$ becomes increasingly large, so the sequence has no limit.

7. $\frac{3}{7}$, $\lim_{n\to\infty} \frac{3n-6}{7n} = \lim_{n\to\infty}\left(\frac{3}{7} - \frac{6}{7} \cdot \frac{1}{n}\right)$
$= \lim_{n\to\infty} \frac{3}{7} - \lim_{n\to\infty} \frac{6}{7} \cdot \lim_{n\to\infty} \frac{1}{n}$
$= \frac{3}{7} - \frac{6}{7} \cdot 0$ or $\frac{3}{7}$

8. $0.\overline{7} = \frac{7}{10} + \frac{7}{100} + \cdots$

$a_1 = \frac{7}{10}$, $r = \frac{1}{10}$

$S_n = \frac{\frac{7}{10}}{1 - \frac{1}{10}}$
$= \frac{7}{9}$

9. $5.\overline{126} = 5 + \frac{126}{1000} + \frac{126}{1{,}000{,}000} + \cdots$

$a_1 = \frac{126}{1000}$, $r = \frac{1}{1000}$

$S_n = 5 + \frac{\frac{126}{1000}}{1 - \frac{1}{1000}}$
$= 5 + \frac{126}{999}$
$= 5\frac{14}{11}$

10. $r = \frac{3}{-6}$ or $-\frac{1}{2}$

$S_n = \frac{-6}{1 - \left(-\frac{1}{2}\right)}$
$= -4$

or $\frac{1}{3}$

$\frac{\frac{3}{4}}{-\frac{1}{3}}$

$\sqrt{3}$

...he sum does not exist since $|r| = |\sqrt{3}| > 1$.

13. $a_1 = 75$, $r = \frac{2}{5}$

$S_n = \frac{75}{1 - \frac{2}{5}}$
$= 125$ m

Pages 781–783 Exercises

14. $\lim_{n\to\infty} \frac{7 - 2n}{5n} = \lim_{n\to\infty}\left(\frac{7}{5} \cdot \frac{1}{n} - \frac{2}{5}\right)$
$= \lim_{n\to\infty} \frac{7}{5} \cdot \lim_{n\to\infty} \frac{1}{n} - \lim_{n\to\infty} \frac{2}{5}$
$= \frac{7}{5} \cdot 0 - \frac{2}{5}$ or $-\frac{2}{5}$

15. $\lim_{n\to\infty} \frac{n^3 - 2}{n^2} = \lim_{n\to\infty}\left(n - \frac{2}{n}\right)$.

$\lim_{n\to\infty} \frac{2}{n} = \lim_{n\to\infty} 2 \cdot \frac{1}{n} = 2 \cdot 0$ or 0, but as n approaches infinity, n becomes increasingly large, so the sequence has no limit.

16. $\lim_{n\to\infty} \frac{6n^2 + 5}{3n^2} = \lim_{n\to\infty}\left(\frac{6}{3} + \frac{5}{3n^2}\right)$
$= \lim_{n\to\infty} 2 + \lim_{n\to\infty} \frac{5}{3} \cdot \lim_{n\to\infty} \frac{1}{n^2}$
$= 2 + \frac{5}{3} \cdot 0$ or 2

17. $\lim_{n\to\infty} \frac{9n^3 + 5n - 2}{2n^3} = \lim_{n\to\infty}\left(\frac{9}{2} + \frac{5}{2n^2} - \frac{1}{n^3}\right)$
$= \lim_{n\to\infty} \frac{9}{2} + \lim_{n\to\infty} \frac{5}{2} \cdot \lim_{n\to\infty} \frac{1}{n^2} - \lim_{n\to\infty} \frac{1}{n^3}$
$= \frac{9}{2} + \frac{5}{2} \cdot 0 - 0$ or $\frac{9}{2}$

18. $\lim_{n\to\infty} \frac{(3n+4)(1-n)}{n^2} = \lim_{n\to\infty} \frac{-3n^2 - n + 4}{n^2}$
$= \lim_{n\to\infty}\left(-3 - \frac{1}{n} + \frac{4}{n^2}\right)$
$= \lim_{n\to\infty}(-3) - \lim_{n\to\infty} \frac{1}{n} + \lim_{n\to\infty} 4 \cdot \lim_{n\to\infty} \frac{1}{n^2}$
$= -3 - 0 + 4 \cdot 0$ or -3

19. Dividing by the highest powered term, n^2, we find $\lim_{n\to\infty} \frac{8 + \frac{5}{n} + \frac{2}{n^2}}{\frac{3}{n^2} + \frac{2}{n}}$ which as n approaches infinity simplifies to $\frac{8 + 0 + 0}{0 + 0} = \frac{8}{0}$. Since this fraction is undefined, the limit does not exist.

20. $\lim_{n\to\infty} \frac{4 - 3n + n^2}{2n^3 - 3n^2 + 5} = \lim_{n\to\infty} \frac{\frac{4}{n^3} - \frac{3n}{n^3} + \frac{n^2}{n^3}}{\frac{2n^3}{n^3} - \frac{3n^2}{n^3} + \frac{5}{n^3}}$

$= \lim_{n\to\infty} \frac{\frac{4}{n^3} - \frac{3}{n^2} + \frac{1}{n}}{2 - \frac{3}{n} + \frac{5}{n^3}}$

$= \frac{\lim_{n\to\infty} 4 \cdot \lim_{n\to\infty} \frac{1}{n^3} - \lim_{n\to\infty} 3 \cdot \lim_{n\to\infty} \frac{1}{n^2} + \lim_{n\to\infty} \frac{1}{n}}{\lim_{n\to\infty} 2 - \lim_{n\to\infty} 3 \cdot \lim_{n\to\infty} \frac{1}{n} + \lim_{n\to\infty} 5 \cdot \lim_{n\to\infty} \frac{1}{n^3}}$

$= \frac{4 \cdot 0 - 3 \cdot 0 + 0}{2 - 3 \cdot 0 + 5 \cdot 0}$ or 0

21. As $n \to \infty$, 3^n becomes increasingly large and thus the value $\frac{1}{3^n}$ becomes smaller and smaller, approaching zero. So the sequence has a limit of zero.

22. Dividing by the highest powered term, n, we find

$\lim_{n\to\infty} \frac{\frac{(-2)^n}{n}}{\frac{4}{n} + 1} = \lim_{n\to\infty} \left(\frac{(-2)^n}{n} \cdot \frac{1}{\frac{4}{n} + 1} \right)$

which as n approaches infinity

$\lim_{n\to\infty} \frac{1}{\frac{4}{n} + 1} = 1$, but $\lim_{n\to\infty} \frac{(-2)^n}{n}$ has no limit since $|-2| > 1$.

23. $\lim_{n\to\infty} \frac{5n + (-1)^n}{n^2} = \lim_{n\to\infty} \frac{5n}{n^2} + \lim_{n\to\infty} \frac{(-1)^n}{n^2}$

$= \lim_{n\to\infty} \frac{5}{n} + \lim_{n\to\infty} \frac{(-1)^n}{n^2}$

$= \lim_{n\to\infty} \frac{(-1)^n}{n^2}$

As n increases, the value of the numerator alternates between -1 and 1. As n approaches infinity, the value of the denominator becomes increasingly large, causing the value of the fraction to become increasingly small. Thus, the terms of the sequence alternate between smaller and smaller positive and negative values, approaching zero. So the sequence has a limit of zero.

24. $0.\overline{4} = \frac{4}{10} + \frac{4}{100} + \ldots$

$a_1 = \frac{4}{10}, r = \frac{1}{10}$

$S_n = \frac{\frac{4}{10}}{1 - \frac{1}{10}}$

$= \frac{4}{9}$

25. $0.\overline{51} = \frac{51}{100} + \frac{51}{10{,}000} + \ldots$

$a_1 = \frac{51}{100}, r = \frac{1}{100}$

$S_n = \frac{\frac{51}{100}}{1 - \frac{1}{100}}$

$= \frac{51}{99}$ or $\frac{17}{33}$

26. $0.\overline{370} = \frac{370}{1000} + \frac{370}{1{,}000{,}000} + \ldots$

$a_1 = \frac{370}{1000}, r = \frac{1}{1000}$

$S_n = \frac{\frac{370}{1000}}{1 - \frac{1}{1000}}$

$= \frac{370}{999}$ or $\frac{10}{27}$

27. $6.\overline{259} = 6 + \frac{259}{1000} + \frac{259}{1{,}000{,}000} + \ldots$

$a_1 = \frac{259}{1000}, r = \frac{1}{1000}$

$S_n = 6 + \frac{\frac{259}{1000}}{1 - \frac{1}{1000}}$

$= 6 + \frac{259}{999}$

$= 6\frac{7}{27}$

28. $0.\overline{15} = \frac{15}{100} + \frac{15}{10{,}000} + \ldots$

$a_1 = \frac{15}{100}, r = \frac{1}{100}$

$S_n = \frac{\frac{15}{100}}{1 - \frac{1}{100}}$

$= \frac{15}{99}$ or $\frac{5}{33}$

29. $0.2\overline{63} = \frac{2}{10} + \frac{63}{1000} + \frac{63}{100{,}000} + \ldots$

$a_1 = \frac{63}{1000}, r = \frac{1}{100}$

$S_n = \frac{1}{5} + \frac{\frac{63}{1000}}{1 - \frac{1}{100}}$

$= \frac{1}{5} + \frac{63}{990}$

$= \frac{29}{110}$

30. The series is geometric, having a common ratio of 0.1. Since this ratio is less than 1, the sum of the series exists and is $\frac{2}{9}$.

31. $r = \frac{12}{16}$ or $\frac{3}{4}$

$S_n = \frac{16}{1 - \frac{3}{4}}$

$= 64$

32. $r = \frac{7.5}{5}$ or 1.5

This series is geometric with a common ratio of 1.5. Since this ratio is greater than 1, the sum of the series does not exist.

33. $r = \frac{5}{10}$ or $\frac{1}{2}$

$S_n = \frac{10}{1 - \frac{1}{2}}$

$= 20$

34. The series is arithmetic, having a general term of $7 - n$. Since $\lim_{n\to\infty} 7 - n$ does not equal zero, this series has no sum.

35. $r = \frac{\frac{1}{4}}{\frac{1}{8}}$ or 2

This series is geometric with a common ratio of 2. Since this ratio is greater than 1, the sum of the series does not exist.

36. $r = \dfrac{\frac{1}{9}}{-\frac{2}{3}}$ or $-\frac{1}{6}$

$S_n = \dfrac{-\frac{2}{3}}{1 - \left(-\frac{1}{6}\right)}$

$= -\frac{4}{7}$

37. $r = \dfrac{\frac{4}{5}}{\frac{6}{5}}$ or $\frac{2}{3}$

$S_n = \dfrac{\frac{6}{5}}{1 - \frac{2}{3}}$

$= 3\frac{3}{5}$

38. $r = \frac{1}{\sqrt{5}}$ or $\frac{\sqrt{5}}{5}$

$S_n = \dfrac{\sqrt{5}}{1 - \frac{\sqrt{5}}{5}}$

$= \dfrac{\sqrt{5}}{1 - \frac{\sqrt{5}}{5}} \cdot \left(\dfrac{1 + \frac{\sqrt{5}}{5}}{1 + \frac{\sqrt{5}}{5}}\right)$

$= \dfrac{\sqrt{5} + 1}{1 - \frac{5}{25}}$

$= \frac{4}{5}(\sqrt{5} + 1)$

39. $r = -\frac{4\sqrt{3}}{8}$ or $-\frac{\sqrt{3}}{2}$

$S_n = \dfrac{8}{1 - \left(-\frac{\sqrt{3}}{2}\right)}$

$= \dfrac{8}{1 + \frac{\sqrt{3}}{2}} \cdot \dfrac{\left(1 - \frac{\sqrt{3}}{2}\right)}{\left(1 - \frac{\sqrt{3}}{2}\right)}$

$= \dfrac{8\left(1 - \frac{\sqrt{3}}{2}\right)}{1 - \frac{3}{4}}$

$= 32\left(1 - \frac{\sqrt{3}}{2}\right)$

$= 32 - 16\sqrt{3}$

40a. $a_1 = 35$ $r = \frac{2}{5}$

$a_2 = 35\left(\frac{2}{5}\right)$ or 14

$a_3 = 14$

$a_4 = 14\left(\frac{2}{5}\right)$ or 5.6

$a_5 = 5.6$

35, 14, 14, 5.6, 5.6

40b. $S_n = 35 + \dfrac{14}{1 - \frac{2}{5}} + \dfrac{14}{1 - \frac{2}{5}}$

$= 35 + \frac{20}{3} + \frac{70}{3}$

$= 81\frac{2}{3}$ m or about 82 m

41a. The limit of a difference equals the difference of the limits only if the two limits exist. Since neither $\lim\limits_{n\to\infty} \frac{n^2}{2n+1}$ nor $\lim\limits_{n\to\infty} \frac{n^2}{2n-1}$ exists, this property of limits does not apply.

41b. $\lim\limits_{n\to\infty} \left(\frac{n^2}{2n+1} - \frac{n^2}{2n-1}\right) = \lim\limits_{n\to\infty} \left[\frac{n^2(2n-1) - n^2(2n+1)}{(2n+1)(2n-1)}\right]$

$= \lim\limits_{n\to\infty} \frac{2n^3 - n^2 - 2n^3 - n^2}{4n^2 - 1}$

$= \lim\limits_{n\to\infty} \frac{-2n^2}{4n^2 - 1}$

$= \lim\limits_{n\to\infty} \dfrac{\frac{-2n^2}{n^2}}{\frac{4n^2}{n^2} - \frac{1}{n^2}}$

$= \lim\limits_{n\to\infty} \dfrac{-2}{4 - \frac{1}{n^2}}$

$= -\frac{1}{2}$

42a. $12 \div 4 = 3$

$r = \sqrt[3]{3}$

$a_4 = a_1 r^{4-1}$

$4 = a_1(\sqrt[3]{3})^3$

$4 = a_1(3)$

$\frac{4}{3} = a_1$

42b. $a_{28} = a_1 r^{28-1}$

$= \frac{4}{3}(\sqrt[3]{3})^{27}$

$= \frac{4}{3}(3^9)$

$= 4(3^8)$

$= 26{,}244$

43. No; if n is even, $\lim\limits_{n\to\infty} \cos \frac{n\pi}{2} = \frac{1}{2}$, but if n is odd, $\lim\limits_{n\to\infty} \cos \frac{n\pi}{2} = -\frac{1}{2}$.

44a. After 2 hours, $\frac{1}{2}D$ exists. After 4 hours, $\frac{1}{2} \cdot \frac{1}{2}D$ or $\frac{1}{4}D$ exists. After 6 hours and before the second dose, $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}D$ or $\frac{1}{8}D$ exists.

44b. $a_1 = D$, $r = \frac{1}{8}$

$S_n = D\dfrac{\left[1 - \left(\frac{1}{8}\right)^n\right]}{1 - \frac{1}{8}}$

$= \frac{8}{7}D\left[1 - \left(\frac{1}{8}\right)^n\right]$

44c. $\lim\limits_{n\to\infty} S_n = S$

$S = \frac{a_1}{1 - r}$

$= \dfrac{D}{1 - \frac{1}{8}}$

$= \frac{8}{7}D$

44d. $350 \geq \frac{8}{7}D$

$306.25 \geq D$

The largest possible dose is 306.25 mg.

45a. A side of the original square measures $\frac{20}{4}$ or 5 feet. Half of 5 feet is $\frac{5}{2}$ feet.

$\left(\frac{5}{2}\right)^2 + \left(\frac{5}{2}\right)^2 = s^2$

$\frac{50}{4} = s^2$

$\frac{5\sqrt{2}}{2} = s$

Perimeter $= 4 \cdot \frac{5\sqrt{2}}{2}$ or $10\sqrt{2}$ feet.