11-7 Modeling Real-World Data with Exponential and Logarithmic Functions

Page 744 Check for Understanding

1. Replace N by $4N_0$ in the equation $N = N_0e^{kt}$, where N_0 is the amount invested and k is the interest rate. Then solve for t.
2. The data should be modeled with an exponential function. The points in the scatter plot approach a horizontal asymptote. Exponential functions have horizontal asymptotes, but logarithmic functions do not.
3. $y = 2e^{(\ln 4)x}$ or $y = 2e^{1.3863x}$; $\ln y = \ln 2 + (\ln 4)x$ or $\ln y =$

11-7
745
7-15, 21

4. $t = \frac{\ln}{0.0}$
≈ 39

6a. $y =$

6b. $y =$
$y =$
$y =$
$y \approx$

6c.
$\ln \frac{}{1}$

$\frac{\ln \frac{}{10.0170}}{-0.0301} \approx x$
$23.08 \approx x$; 23.08 min

Pages 745–748 Exercises

7. $t = \frac{\ln 2}{0.0225}$
≈ 30.81 yr

8. $t = \frac{\ln 2}{0.05}$
≈ 13.86 yr

9. $t = \frac{\ln 2}{0.07125}$
≈ 9.73

10. exponential; the graph has a horizontal asymptote
11. logarithmic; the graph has a vertical asymptote
12. logarithmic; the graph has a vertical asymptote
13. exponential; the graph has a horizontal asymptote

14a. $y = 4.7818(1.7687)^x$

14b. $y = 4.7818(1.7687)^x$
$y = 4.7818(e^{\ln 1.7687})^x$
$y = 4.7818e^{(\ln 1.7687)x}$
$y = 4.7818e^{0.5702x}$

14c. Use $t = \frac{\ln 2}{k}$; $k = 0.5702$.
$t = \frac{\ln 2}{0.5702}$
≈ 1.215 hr

15a. $y = 1.0091(0.9805)^x$

15b. $y = 1.0091(0.9805)^x$
$y = 1.0091(e^{\ln 0.9805})^x$
$y = 1.0091e^{(\ln 0.9805)x}$
$y = 1.0091e^{-0.0197x}$

15c. $0.415 = 1.0091e^{-0.0197x}$
$\ln \frac{0.415}{1.0091} = -0.0197x$
$\frac{\ln \frac{0.415}{1.0091}}{-0.0197} = x$
$45.10 \approx x$
$45.10 - 10 = 35.10$ min

16a. $y = 2137.5192(1.0534)^x$

16b. $y = 2137.5192(1.0534)^x$
$y = 2137.5192(e^{\ln 1.0534})^x$
$y = 2137.5192e^{(\ln 1.0534)x}$
$y = 2137.5192e^{0.0520x}$

16c. $2631.74 = 2137.52e^{4r}$
$\ln \frac{2631.74}{2137.52} = 4r$
$\frac{\ln \frac{2631.74}{2137.52}}{4} = r$
$0.0520 \approx r$; 5.2%

17. $y = 40 + 14.4270 \ln x$

18a. $y = -826.4217 + 520.4168 \ln x$

18b. The year 1960 would correspond to $x = 0$ and $\ln 0$ is undefined.

19. Take the square root of each side.
$y = cx^2$
$\sqrt{y} = \sqrt{cx^2}$
$\sqrt{y} = \sqrt{c}x$

20a. $1034.34 = 1000(1 + r)^1$
$1.03034 = 1 + r$
$0.03034 \approx r$; 3.034%

20b. $y = 1000.0006(1.0303)^x$

20c. $y = 1000.0006(1.0303)^x$
$y = 1000.0006(e^{\ln 1.0303})^x$
$y = 1000.0006e^{(\ln 1.0303)x}$
$y = 1000.0006e^{0.0299x}$

20d. $1030.34 = 1000e^r$
$\ln \frac{1030.34}{1000} = r$
$0.0299 \approx r$; 2.99%

21a.

x	0	50	100	150	190	200
$\ln y$	1.81	2.07	3.24	3.75	4.25	4.38

21b. $\ln y = 0.0136x + 1.6889$

21c. $\ln y = 0.0136x + 1.6889$
$y = e^{0.0136x+1.6889}$

21d. $y = e^{0.0136(225)+1.6889}$
≈ 115.4572
115.5 persons per square mile

22a. The graph appears to have a horizontal asymptote at $y = 2$, so you must subtract 2 from each y-value before a calculator can perform exponential regression.

22b. $y = 2 + 1.0003(2.5710)^x$

23a. $\ln y$ is a linear function of $\ln x$.
$y = cx^a$
$\ln y = \ln(cx^a)$
$\ln y = \ln c + \ln x^a$
$\ln y = \ln c + a \ln x$