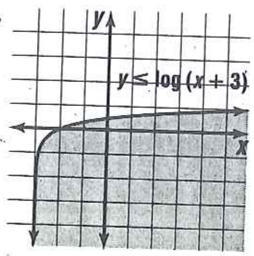
11-5 Common Logarithms

Page 730 Check for Understanding

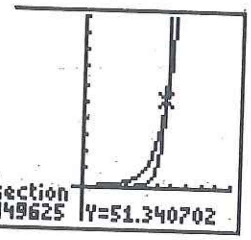
1. $\log 1 = 0$ means $\log_{10} 1 = 0$. So, $10^0 = 1$.
   $\log 10 = 1$ means $\log_{10} 10 = 1$. So, $10^1 = 10$.
2. Write the number in scientific notation. T
   exponent of the power of 10 is the c
3. antilog $2.835 = 10^{2.835} = 685$
4. $\log 15 = 1.1761$
   $\log 5 = 0.6990$
   $\log 3 = 0.4771$
   $\log 5 + \log 3 = 0.6990 + 0.4771 =$
5. $\log 80{,}000 = \log (10{,}000 \times 8)$
   $= \log 10^4 + \log 8$
   $= 4 + 0.9031$
   $= 4.9031$
6. $\log 0.003 = \log (0.001 \times 3)$
   $= \log 10^{-3} + \log 3$
   $= -3 + 0.4771$
   $= -2.5229$
7. $\log 0.0081 = \log (0.0001 \times 3^4)$
   $= \log 10^{-4} + 4 \log 3$
   $= -4 + 4(0.4771)$
   $= -2.0915$
8. 2.6274
9. 74,816.95
10.



11. $\log_{12} 18 = \frac{\log 18}{\log 12}$
    ≈ 1.1632
12. $\log_8 15 = \frac{\log 15}{\log 8}$
    ≈ 1.3023
13. $2.2^{x-5} = 9.32$
    $(x - 5) \log 2.2 = \log 9.32$
    $(x - 5) = \frac{\log 9.32}{\log 2.2}$
    $x \approx 7.83$
14. $6^{x-2} = 4^x$
    $(x - 2) \log 6 = x \log 4$
    $x \log 6 - 2 \log 6 = x \log 4$
    $-2 \log 6 = x \log 4 - x \log 6$
    $-2 \log 6 = x (\log 4 - \log 6)$
    $\frac{-2 \log 6}{\log 4 - \log 6} = x$
    $8.84 \approx x$
15. $4.3^x < 76.2$
    $x \log 4.3 < \log 76.2$
    $x < \frac{\log 76.2}{\log 4.3}$
    $x < 2.97$

11.5
730
19-39 odd
40-43
48-52
55-57

16. $3^{x-3} \geq 2 \sqrt[4]{4^{x-1}}$
    $3^{x-3} \geq 2\left(4^{\frac{x-1}{4}}\right)$
    $(x - 3) \log 3 \geq \log 2 + \frac{x-1}{4} \log 4$
    $(4x - 12)\log 3 \geq 4 \log 2 + (x - 1) \log 4$
    $x \log 3 - 12 \log 3 \geq 4 \log 2 + x \log 4 - \log 4$
    $x \log 3 - x \log 4 \geq 4 \log 2 - \log 4 + 12 \log 3$
    $(4 \log 3 - \log 4) \geq 4 \log 2 - \log 4 + 12 \log 3$
    $x \geq \frac{4 \log 2 - \log 4 + 12 \log 3}{4 \log 3 - \log 4}$
    $x \geq 4.84$



[10, 10] scl:1 by [−20, 100] scl:10
5.5850

18a. $R = \log \left(\frac{200}{1.6}\right) + 4.2$
     $= 6.3$

18b. 10 times; According to the definition of logarithms, R in the equation $R = \log \left(\frac{a}{T}\right) + B$ is an exponent of the base of the logarithm, 10. 10^5 is ten times greater than 10^4.

Pages 730–732 Exercises

19. $\log 4000{,}000 = \log (100{,}000 \times 4)$
    $= \log 100{,}000 + \log 4$
    $= 5 + 0.6021$
    $= 5.6021$
20. $\log 0.00009 = \log (0.00001 \times 9)$
    $= \log 0.00001 + \log 9$
    $= -5 + 0.9542$
    $= -4.0458$
21. $\log 1.2 = \log (0.1 \times 12)$
    $= \log 0.1 + \log 12$
    $= -1 + 1.0792$
    $= 0.0792$
22. $\log 0.06 = \log \left(0.01 \times \frac{12}{2}\right)$
    $= \log 0.01 + \log \frac{12}{4^{\frac{1}{2}}}$
    $= \log 0.01 + \log 12 - \frac{1}{2} \log 4$
    $= -2 + 1.0792 - \frac{1}{2}(0.6021)$
    $= -1.2218$
23. $\log 36 = \log (4 \times 9)$
    $= \log 4 + \log 9$
    $= 0.6021 + 0.9542$
    $= 1.5563$
24. $\log 108{,}000 = \log (1000 \times 12 \times 9)$
    $= \log 1000 + \log 12 + \log 9$
    $= 3 + 1.0792 + 0.9542$
    $= 5.0334$

25. $\log 0.0048 = \log(0.0001 \times 12 \times 4)$
$= \log 0.0001 + \log 12 + \log 4$
$= -4 + 1.0792 + 0.6021$
$= -2.3188$

26. $\log 4.096 = \log(0.001 \cdot 4^6)$
$= \log 0.001 + 6 \log 4$
$= -3 + 6(0.6021)$
$= 0.6124$

27. $\log 1800 = \log\left(100 \times 9 \times 4^{\frac{1}{2}}\right)$
$= \log 100 + \log 9 + \frac{1}{2}\log 4$
$= 2 + 0.9542 + \frac{1}{2}(0.6021)$
$= 3.2553$

28. 1.9921

29. 2.9515

30. 0.871

31. 2.001

32. 3.2769

33. 2.1745

34. $\log_2 8 = \frac{\log 8}{\log 2}$
$= 3$

35. $\log_5 625 = \frac{\log 625}{\log 5}$
$= 4$

36. $\log_6 24 = \frac{\log 24}{\log 6}$
≈ 1.7737

37. $\log_7 4 = \frac{\log 4}{\log 7}$
≈ 0.7124

38. $\log_{6.5} 0.0675 = \frac{\log 0.0675}{\log 0.5}$
≈ 3.8890

39. $\log_{\frac{1}{2}} 15 = \dfrac{\log 15}{\log \frac{1}{2}}$

40. $2^x = 95$
$x \log 2 = \log 95$
$x = \frac{\log 95}{\log 2}$
$x \approx 6.5699$

41. $5x = 4^{x+3}$
$x \log 5 = (x+3)\log 4$
$x \log 5 = x \log 4 + 3 \log 4$
$x(\log 5 - \log 4) = 3 \log 4$
$x = \frac{3 \log 4}{\log 5 - \log 4}$
$x \approx 18.6377$

42. $\frac{1}{3}\log x = \log 8$
$x^{\frac{1}{3}} = 8$
$x \approx 512$

43. $0.16^{4+3x} = 0.3^{8-x}$
$(4+3x)\log 0.16 = (8-x)\log 0.3$
$4 \log 0.16 + 3x \log 0.16 = 8 \log 0.3 - x \log 0.3$
$3x \log 0.16 + x \log 0.3 = 8 \log 0.3 - 4 \log 0.16$
$x(3 \log 0.16 + \log 0.3) = 8 \log 0.3 - 4 \log 0.16$
$x = \frac{8 \log 0.3 - 4 \log 0.16}{3 \log 0.16 + \log 0.3}$
$x \approx 0.3434$

44. $4 \log(x+3) = 9$
$\log(x+3) = \frac{9}{4}$
$(x+3) = \text{antilog } \frac{9}{4}$
$x = \text{antilog } \frac{9}{4} - 3$
$x \approx 174.8297$

$10^{9/4} = x+3$
or
$\log(x+3)^4 = 9$
$10^{9(1/4)} = (x+3)^{4(1/4)}$

45. $0.25 = \log 16^x$
$0.25 = x \log 16$
$x = \frac{0.25}{\log 16}$
$x \approx 0.2076$

46. $3^{x-1} \le 2^{x-7}$
$(x-1)\log 3 \le (x-7)\log 2$
$x \log 3 - \log 3 \le x \log 2 - 7 \log 2$
$x \log 3 - x \log 2 \le \log 3 - 7 \log 2$
$x(\log 3 - \log 2) \le \log 3 - 7 \log 2$
$x \le \frac{\log 3 - 7 \log 2}{\log 3 - \log 2}$
$x = -9.2571$

47. $\log_x 6 > 1$
$\frac{\log 6}{\log x} > 1$
$\log 6 > \log x$
$6 > x$

change of base
$x^1 > 6$ =)

When $x = 1$, $\log 1 = 0$, which means $\frac{\log 6}{\log x}$ is undefined. When $x < 1$, $\frac{\log 6}{\log x}$ is negative, which is not greater than 1. So, x must also be greater than 1. Therefore, $1 < x < 6$.

48. $4^{2x-5} \le 3^{x-3}$
$(2x-5)\log 4 \le (x-3)\log 3$
$2x \log 4 - 5 \log 4 \le x \log 3 - 3 \log 3$
$2x \log 4 - x \log 3 \le 5 \log 4 - 3 \log 3$
$x(2 \log 4 - \log 3) \le 5 \log 4 - 3 \log 3$
$x \le \frac{5 \log 4 - 3 \log 3}{2 \log 4 - \log 3}$
$x \le 2.1719$

49. $0.5^{2x-4} \le 0.1^{5-x}$
$(2x-4)\log 0.5 \le (5-x)\log 0.1$
$2x(\log 0.5) - 4 \log 0.5 \le 5 \log 0.1 - x \log 0.1$
$2x \log 0.5 + x \log 0.1 \le 5 \log 0.1 + 4 \log 0.5$
$x(2 \log 0.5 + \log 0.1) \le 5 \log 0.1 + 4 \log 0.5$
$x \ge \frac{5 \log 0.1 + 4 \log 0.5}{2 \log 0.5 + \log 0.1}$

Change inequality sign because (2 log 0.5 + log 0.1) is negative.

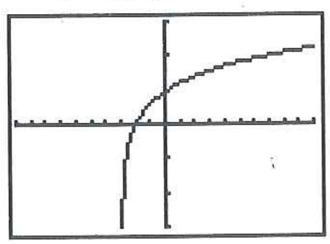
$x \ge 3.8725$

50. $\log_2 x = -3$
$x = 2^{-3}$
$x = 0.1250$

51. $x < \frac{\log 52.7}{\log 3}$
$x < 3.6087$

$3^x < 5.27$
$x \log 3 < \log 5.27$
$x < \frac{\log 52.7}{\log 3}$
$x < 3.609$

52.



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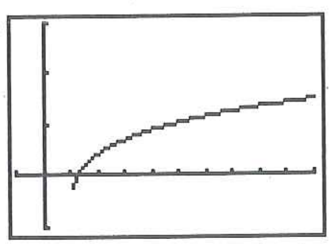
53.



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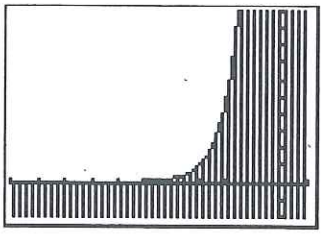
54.



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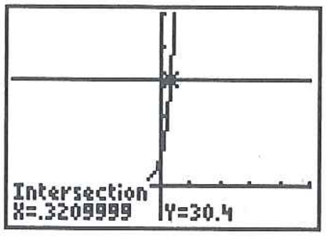
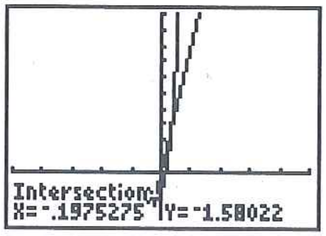
55.



$x \approx 0.3210$

$[-5, 5]$ scl:1 by $[-10, 50]$ scl:10

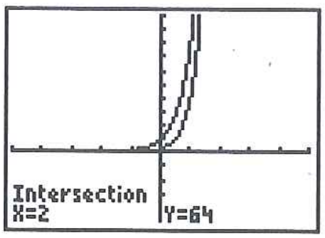
56.



$x \approx -0.1975$

$[-5, 5]$ scl:1 by $[-3, 10]$ scl:1

57.



$x = 2$

$[-5, 5]$ scl:1 by $[-5, 10]$ scl:1

58a. $h = -\frac{100}{9} \log \frac{10.3}{14.7}$

≈ 1.7 mi

58b.

$$\begin{aligned} 4.3 &= -\frac{100}{9} \log \frac{P}{14.7} \\ -0.3870 &= \log P - \log 14.7 \\ -0.3870 + \log 14.7 &= \log P \\ 0.7803 &\approx \log P \\ 6.03 &\approx P;\ 6 \text{ psi} \end{aligned}$$

take anti log (

$-.387 = \log \frac{P}{14.7}$

$.41 = \frac{P}{14.7}$

59a. $M = 5.3 + 5 + 5 \log 0.018$

≈ 1.58

59b.

$$\begin{aligned} 5.3 &= 8.6 + 5 + 5 \log P \\ -8.3 &= 5 \log P \\ -1.66 &= \log P \\ 0.0219 &\approx P \end{aligned}$$

60a. $q = \left(\frac{1}{2}\right)^{0.8^9}$

$= \left(\frac{1}{2}\right)^{0.1342}$

$= 0.9112$

\$91,116

60b.

$$\begin{aligned} 0.9535 &= \left(\tfrac{1}{2}\right)^{0.8^t} \\ \log 0.9535 &= 0.8^t \log \tfrac{1}{2} \\ \frac{\log 0.9535}{\log \frac{1}{2}} &= 0.8^t \\ \log\left[\frac{\log 0.9535}{\log \frac{1}{2}}\right] &= t \log 0.8 \\ 12.0016 &\approx t \end{aligned}$$

12 years

$= .68695$

$t \log .8 = \log .68695$

$t = \frac{\log .68695}{\log .8}$

61. Sample answer: x is between 2 and 3 because 372 is between 100 and 1000, and $\log 100 = 2$ and $\log 1000 = 3$.

62a. $L = 10 \log \frac{1}{1.0 \times 10^{-12}}$

$= 10(\log 1 - \log (1.0 \times 10^{-12}))$

$= 120$ dB

62b.

$$\begin{aligned} 20 &= \log \frac{I}{1.0 \times 10^{-12}} \\ 2 &= \log I - \log (1.0 \times 10^{-12}) \\ 2 &= \log I + 12 \\ -10 &= \log I \end{aligned}$$

$1 \times 10^{-10} = I$; 1×10^{-10} W/m^2

antilog)

63. Use $N = N_0\left(\frac{1}{2}\right)^t$.

$N = 630$ micrograms $= 63 \times 10^{-4}$ gram

$N_0 = 1$ milligram $= 1.0 \times 10^{-3}$ gram

$6.3 \times 10^{-4} = (1.0 \times 10^{-3})\left(\frac{1}{2}\right)^t$

$\log \frac{6.3 \times 10^{-4}}{1.0 \times 10^{-3}} = t \log \frac{1}{2}$

$0.6666 \approx t$

$0.6666 \times 5730 \approx 3819$ yr

64. $\log_a y = \log_a P - \log_a q + \log_a r$

$\log_a y = \log_a \frac{p}{q} + \log_a r$

$\log_a y = \log_a \frac{pr}{q}$

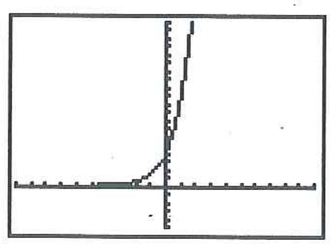
$y = \frac{pr}{q}$

65. $\log_x 243 = 5$

$x^5 = 243$

$x = 3$

66.



increasing from $-\infty$ to ∞

67. $(a^4b^2)^{\frac{1}{3}}c^{\frac{2}{3}} = (a^4)^{\frac{1}{3}}(b^2)^{\frac{1}{3}}(c^2)^{\frac{1}{3}}$

$= a\sqrt[3]{ab^2c^2}$

68. $(5)^2 + (0)^2 + D(5) + E(0) + F = 0$

$5D + F + 25 = 0$

$(1)^2 + (-2)^2 + D(1) + E(-2) + F = 0$

$D - 2E + F + 5 = 0$

$(4)^2 + (-3)^2 + D(4) + E(-3) + F = 0$

$4D - 3E + F + 25 = 0$