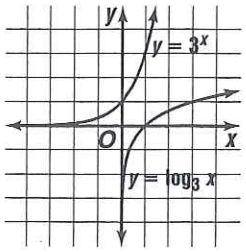
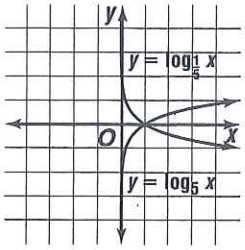
11-4 Logarithmic Functions

Pages 722–723 Check for Understanding

1. 

$y = 3^x$ and $\log_3 x$ are similar in that they are both continuous, one-to-one, increasing and inverses.
$y = 3^x$ and $\log_3 x$ are not similar in that they are inverses. The domain of one is the range of another and the range of one is the domain of the other. $y = 3^x$ has a y-intercept and a horizontal asymptote whereas $y = \log_3 x$ has a x-intercept and a vertical asymptote.

2. Let $b^x = m$, then $\log_b m = x$.
$$(b^x)^p = m^p$$
$$b^{xp} = m^p$$
$$\log_b b^{xp} = \log_b m^p$$
$$xp = \log_b m^p$$
$$p \log_b m = \log_b m^p$$

3. 

$\text{Log}_5 x$ is an increasing function and $\log_{\frac{1}{5}} x$ decreasing function.

4. Sean is correct. The product property states $\log_b mn = \log_b m + \log_b n$.

5. In half-life applications $r = -\frac{1}{2}$. So, $(1 + r)$ becomes $\left(1 - \frac{1}{2}\right)$ or $\left(\frac{1}{2}\right)$. Thus, the formula $N = N_0(1 + r)^t$ becomes $N = N_0\left(\frac{1}{2}\right)^t$.

6. $9^{\frac{3}{2}} = 27$

7. $\left(\frac{1}{25}\right)^{-\frac{1}{2}} = 5$

8. $\log_7 y = -6$

9. $\log_8 \frac{1}{4} = -\frac{2}{3}$

10. $\log_2 \frac{1}{16} = x$
$$2^x = \frac{1}{16}$$
$$2^x = 2^{-4}$$
$$x = -4$$

11. $\log_{10} 0.01 = x$
$$10^x = 0.01$$
$$10^x = 10^{-2}$$
$$x = -2$$

12. $\log_7 \frac{1}{343} = x$
$$7^x = \frac{1}{343}$$
$$7^x = 7^{-3}$$
$$x = -3$$

13. $\log_2 x = 5$
$$2^5 = x$$
$$32 = x$$

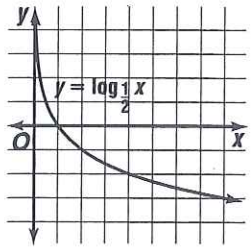
14. $\log_7 n = \frac{2}{3} \log_7 8$
$$\log_7 n = \log_7 8^{\frac{2}{3}}$$
$$n = 8^{\frac{2}{3}}$$
$$n = 4$$

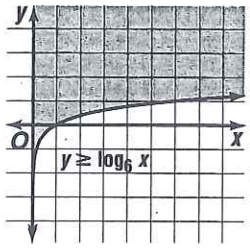
15. $\log_6 (4x + 4) = \log_6 64$
$$4x + 4 = 64$$
$$x = 15$$

16. $2 \log_6 4 - \frac{1}{4} \log_6 16 = \log_6 x$
$$\log_6 4^2 - \log_6 16^{\frac{1}{2}} = \log_6 x$$
$$\log_6 \frac{4^2}{16^{\frac{1}{4}}} = \log_6 x$$
$$\frac{4^2}{16^{\frac{1}{4}}} = x$$
$$x = 8$$

17.

x	y
1	0
2	−1
4	−2





7.4
p 723
21-39 odd
41-53
55

h

4) $\log_4 1024 = x$
$$4^x = 1024$$
$$2^{2x} = 2^{10}$$
$$2x = 10$$
$$x = 5$$

Pages 723–725 Exercises

20. $27^{\frac{1}{3}} = 3$

21. $16^{\frac{1}{2}} = 4$

22. $7^{-4} = \frac{1}{2401}$

23. $4^{\frac{5}{2}} = 32$

24. $e^x = 65.98$

25. $\left(\sqrt{6}\right)^4 = 36$

26. $\log_{81} 9 = \frac{1}{2}$

27. $\log_{36} 216 = \frac{3}{2}$

28. $\log_{\frac{1}{8}} 512 = -3$

29. $\log_6 \frac{1}{36} = -2$

30. $\log_{16} 1 = 0$

31. $\log_x 14.36 = 1.238$

32. $\log_8 64 = x$
$8^x = 64$
$8^x = 8^2$
$x = 2$

33. $\log_{125} 5 = x$
$125^x = 5$
$(5^3)^x = 5^1$
$3x = 1$
$x = \frac{1}{3}$

34. $\log_2 32 = x$
$2^x = 32$
$2^x = 2^5$
$x = 5$

35. $\log_4 128 = x$
$4^x = 128$
$2^{2x} = 2^7$
$2x = 7$
$x = \frac{7}{2}$ or 3.5

36. $\log_9 9^6 = x$
$9^x = 9^6$
$x = 6$

37. $\log_{49} 343 = x$
$49^x = 343$
$7^{2x} = 7^3$
$2x = 3$
$x = \frac{3}{2}$ or 1.5

38. $\log_8 16 = x$
$8^x = 16$
$2^{3x} = 2^4$
$3x = 4$
$x = \frac{4}{3}$

39. $\log_{\sqrt{8}} 4096 = x$
$\left(\sqrt{8}\right)^x = 4096$
$8^{\frac{x}{2}} = 8^4$
$\frac{x}{2} = 4$
$x = 8$

40. $10^{4 \log_{10} 2} = x$
$10^{\log_{10} 2^4} = x$
$2^4 = x$
$16 = x$

41. $\log_x 49 = 2$
$x^2 = 49$
$x = 7$

42. $\log_3 3x = \log_3 36$
$3x = 36$
$x = 12$

43. $\log_6 x + \log_6 9 = \log_6 54$
$\log_6 9x = \log_6 54$
$9x = 54$
$x = 6$

44. $\log_8 48 - \log_8 w = \log_8 6$
$\log_8 \frac{48}{w} = \log_8 6$
$\frac{48}{w} = 6$
$6w = 48$
$w = 8$

45. $\log_6 216 = x$
$6^x = 216$
$6^x = 6^3$
$x = 3$

46. $\log_5 0.04 = x$
$5^x = 0.04$
$5^x = 5^{-2}$
$x = -2$

47. $\log_{10} \sqrt[3]{10} = x$
$10^x = \sqrt[3]{10}$
$10^x = 10^{\frac{1}{3}}$
$x = \frac{1}{3}$

48. $\log_{12} x = \frac{1}{2} \log_{12} 9 + \frac{1}{3} \log_{12} 27$
$\log_{12} x = \log_{12} 9^{\frac{1}{2}} + \log_{12} 27^{\frac{1}{3}}$
$\log_{12} x = \log_{12} 9^{\frac{1}{2}} \cdot 27^{\frac{1}{3}}$
$x = 9^{\frac{1}{2}} \cdot 27^{\frac{1}{3}}$
$x = 3 \cdot 3$
$x = 9$

49. $\log_5 (x + 4) + \log_5 8 = \log_5 64$
$\log_5 (x + 4)(8) = \log_5 64$
$(x + 4)(8) = 64$
$x + 4 = 8$
$x = 4$

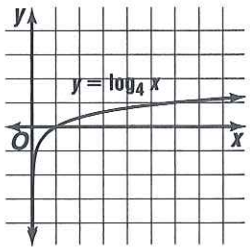
50. $\log_4 (x - 3) + \log_4 (x + 3) = 2$
$\log_4 (x - 3)(x + 3) = 2$
$4^2 = (x - 3)(x + 3)$
$16 = x^2 - 9$
$25 = x^2$
$5 = x$

51. $\frac{1}{2}(\log_7 x + \log_7 8) = \log_7 16$
$\frac{1}{2}(\log_7 8x) = \log_7 16$
$\log_7 (8x)^{\frac{1}{2}} = \log_7 16$
$(8x)^{\frac{1}{2}} = 16$
$8x = 256$
$x = 32$

52. $2 \log_5 (x - 2) = \log_5 36$
$\log_5 (x - 2)^2 = \log_5 36$
$(x - 2)^2 = 36$
$x - 2 = 6$
$x = 8$

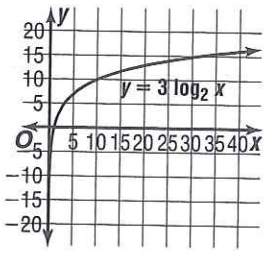
53.

x	y
1	0
2	$\frac{1}{2}$
4	1



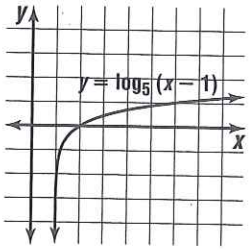
54.

x	y
1	0
2	3
4	6



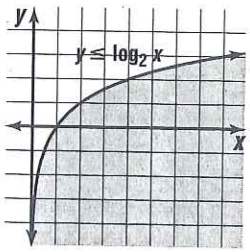
55.

x	y
2	0
6	1



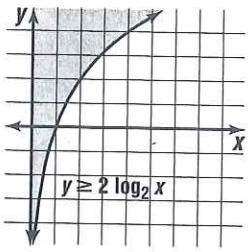
56.

x	y
1	0
2	1
4	2



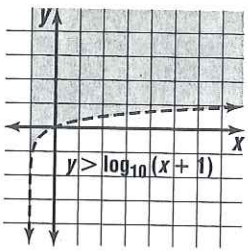
57.

x	y
1	0
2	2
4	4



58.

x	y
0	0
9	1



59. Use $N = N_0(1 + r)^t$; $r = 1$ since the rate of growth is 100% every t time periods.

$64{,}000 = 1000\,(1 + 1)^t$

$64 = 2^t$

$\log_2 2^6 = \log_2 2^t$

$6 = t$

$t \cdot 15 = 90$ min

60. All powers of 1 are 1, so the inverse of $y = 1^x$ is not a function.

61. Let $\log_b m = x$ and $\log_b n = y$.

So, $b^x = m$ and $b^y = n$.

$\frac{m}{n} = \frac{b^x}{b^y} = b^{x-y}$

$\frac{m}{n} = b^{x-y}$

$\log_b \frac{m}{n} = x - y$

$\log_b \frac{m}{n} = \log_b m - \log_b n$

62a. $5000 \geq 2500\left(1 + \frac{r}{4}\right)^{4 \cdot 10}$

$2^7 \geq \left(1 + \frac{r}{4}\right)^{40}$

62b. $2 = \left(1 + \frac{r}{4}\right)^{40}$

$2^{\frac{1}{40}} = \left[\left(1 + \frac{r}{4}\right)^{40}\right]^{\frac{1}{40}}$

$1.0175 \approx 1 + \frac{r}{4}$

$0.0699 \approx r$

6.99%

62c. $2 = \left(1 + \frac{r}{4}\right)^{28}$

$2^{\frac{1}{28}} = 1 + \frac{r}{4}$

$1.0251 \approx 1 + \frac{r}{4}$

$0.1004 \approx r$

10.04%

63a. $n = \log_2 \frac{1}{\frac{1}{4}}$

$n = \log_2 4$

$2^n = 4$

$n = 2$

63b. $3 = \log_2 \frac{1}{p}$

$2^3 = \frac{1}{p}$

$8 = p^{-1}$

$\frac{1}{8} = p$

less light; $\frac{1}{8}$

64. Let $y = \log_a x$, so $x = a^y$.

$x = a^y$

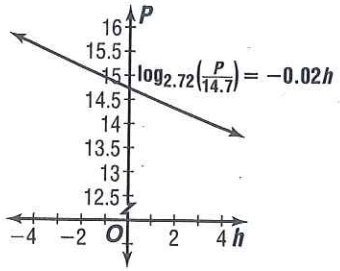
$\log_b x = \log_b a^y$

$\log_b x = y \log_b a$

$y = \frac{\log_b x}{\log_b a}$

$\log_a x = \frac{\log_b x}{\log_b a}$

65a.



65b. $\log_{2.72} \frac{P}{14.7} = -0.02(1)$

$\frac{P}{14.7} = 2.72^{-0.02}$

$P = 14.7\,(272^{-0.02})$

≈ 14.4 psi

65c. $\log_{2.72} \frac{P}{14.7} = -0.02(-6.8)$

$P = 14.7(2.72^{0.136})$

≈ 16.84 psi

66. $6.8 = 38\left(\frac{1}{2}\right)^t$

$\frac{6.8}{38} = \left(\frac{1}{2}\right)^t$

$\log \frac{6.8}{38} = \log \left(\frac{1}{2}\right)^t$

$\log \frac{6.8}{38} = t \log \frac{1}{2}$

$t = \frac{\log \frac{6.8}{38}}{\log \frac{1}{2}}$

$t = 2.5$

$2.5 \cdot 3.82 = 9.55$

about 9 days